Day 229 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Linear inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $-5 < 5x - 3 < 9$.

Subtract -3 from each part: $-2 < 5x < 12$

Divide each part by 5: $-\frac{2}{5} < x < \frac{12}{5}$

Answer: $-\frac{2}{5} < x < \frac{12}{5}$

2 Arc length and sector area (radians) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A sector of a circle has area 19.8 cm^2 and angle 1.1 radians.
Find (a) the radius, (b) the perimeter of the sector.

$\frac{1}{2}r^2\theta = 19.8$, so $r^2 = \frac{2 \times 19.8}{1.1} = 36$, $r = 6$

Perimeter $= 2r + r\theta = 12 + 6.6 = 18.6$ cm

Answer: (a) 6 cm (b) 18.6 cm

3 Constant acceleration with vectors (AQA, Edexcel, OCR, MEI, Eduqas)

A particle has velocity $(5\mathbf{i} + 6\mathbf{j})$ m s^{-1} at time $t = 0$ and moves with constant acceleration $(3\mathbf{i} + 3\mathbf{j})$ m s^{-2}.
Find its velocity and speed when $t = 5$.

$v = u + at = (5\mathbf{i} + 6\mathbf{j}) + 5(3\mathbf{i} + 3\mathbf{j})$

$= 20\mathbf{i} + 21\mathbf{j}$

Speed $= \sqrt{(20^2 + 21^2)}$

Answer: $\mathbf{v} = (20\mathbf{i} + 21\mathbf{j})$ m s^{-1}; speed 29 m s^{-1}

4 Integrating trigonometric functions using identities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the exact value of

$\int_0^{\frac{\pi}{2}} (3\cos^2 x - 1)\,dx$.

Use $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$

$\int (3\cos^2 x - 1)\,dx = \frac{x}{2} + \frac{3\sin 2x}{4}$

$= \left[\frac{x}{2} + \frac{3\sin 2x}{4}\right]_0^{\frac{\pi}{2}}$

$= \frac{\pi}{4}$

Answer: $\frac{\pi}{4}$

5 Parametric curve meeting a line (AQA, Edexcel, OCR, MEI, Eduqas)

A curve has parametric equations $x = 2t - 1$, $y = -t^2 + 4t - 3$.
The line $y = -x + 4$ meets the curve at two points.
Find the coordinates of these points.

Substitute x and y into the line: $-t^2 + 4t - 3 = -2t + 5$

$-t^2 + 6t - 8 = 0$, so $-(t - 4)(t - 2) = 0$

$t = 2$ or $t = 4$, giving the points (3, 1) and (7, −3)

Answer: (3, 1) and (7, −3)

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6 Rough table and a hanging particle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A block of mass 5 kg lies on a rough horizontal table (coefficient of friction 0.3). It is joined by a light string over a smooth pulley to a particle of mass 5 kg hanging freely.

Take g = 9.8 m s^{-2}.

The system is released. Find the acceleration and the tension in the string.

Block: $T - 0.3 \times 5\,g = 5\,a$

Hanging: $5\,g - T = 5\,a$

Add: $5\,g - 1.5\,g = 10\,a$

$a = 3.43$

$T = 5(9.8 - 3.43)$

Answer: a = 3.43 m s^{-2}; T = 31.85 N

7 Standardising to compare (All boards)

Marks are normal: Biology N(60, 8^2), Chemistry N(72, 9^2).

Anna scores 64 in Biology and 90 in Chemistry. Which is her better performance, relatively?

z-score in Biology = $\frac{64 - 60}{8} = 0.500$

z-score in Chemistry = $\frac{90 - 72}{9} = 2.00$

The higher z-score is in Chemistry

Answer: Chemistry

8 Proof by exhaustion (AQA, Edexcel, OCR, MEI, Eduqas)

Prove, by exhaustion, that a cube number cannot be written in the form 7k + 5, where k is an integer.

Write the integer as n = 7q + j, where j = 0, 1, ..., 6.

n^3 leaves the same remainder as j^3 when divided by 7.

j = 0, 1, 2, 3, 4, 5, 6 give remainders 0, 1, 1, 6, 1, 6, 6.

The only possible remainders are 0, 1, 6, never 5.

Answer: possible remainders 0, 1, 6, never 5

9 Probability without replacement (All boards)

A bag has 4 white and 6 black sweets; two are taken without replacement.

(a) Find P(exactly one white).

(b) Find P(at least one white).

(a) P(white then black) + P(black then white) = $2 \times \frac{4}{10} \times \frac{6}{9} = \frac{8}{15}$

(b) 1 − P(both black) = $1 - \frac{6}{10} \times \frac{5}{9} = 1 - \frac{1}{3} = \frac{2}{3}$

Answer: (a) $\frac{8}{15}$ (b) $\frac{2}{3}$

10 Inequalities with exponents and logs (All boards)

£1000 is invested at 5% compound interest per year.

Find the least whole number of years for the investment to exceed £5000.

$1000 \times 1.05^n > 5000$, so $1.05^n > 5$

$n > \frac{\ln 5}{\ln 1.05} = 32.99$

The least whole number is n = 33

Answer: 33 years