Day 228 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Position vectors and dividing a line in a ratio (All boards)

A has position vector $\begin{pmatrix} 5 \\ -2 \end{pmatrix}$ and B has position vector $\begin{pmatrix} -4 \\ 4 \end{pmatrix}$.

The point P lies on AB with AP : PB = 2 : 1.

Find the position vector of P.

$\text{AB} = \begin{pmatrix} -9 \\ 6 \end{pmatrix}$

$\text{AP} = \frac{2}{3}\text{AB} = \begin{pmatrix} -6 \\ 4 \end{pmatrix}$

$\text{OP} = \text{OA} + \text{AP} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$

Answer: $\begin{pmatrix} \mathbf{-1} \\ \mathbf{2} \end{pmatrix}$

2 Linear inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $-7 < 4x + 2 \leq 2$.

Subtract 2 from each part: $-9 < 4x \leq 0$

Divide each part by 4: $-\frac{9}{4} < x \leq 0$

Answer: $\mathbf{-\frac{9}{4} < x \leq 0}$

3 Constant acceleration with vectors (AQA, Edexcel, OCR, MEI, Eduqas)

A particle has velocity $(3\mathbf{i} + 4\mathbf{j})$ m s^{-1} at time t = 0 and moves with constant acceleration $(-2\mathbf{j})$ m s^{-2}.

Find its velocity and speed when t = 4.

$\mathbf{v} = \mathbf{u} + \mathbf{at} = (3\mathbf{i} + 4\mathbf{j}) + 4(-2\mathbf{j})$

$= 3\mathbf{i} - 4\mathbf{j}$

Speed $= \sqrt{(3^2 + -4^2)}$

Answer: $\mathbf{v} = (3\mathbf{i} - 4\mathbf{j})$ m s^{-1}; speed 5 m s^{-1}

4 Sigma notation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Evaluate $\sum_{r=1}^{5} 2 \times 2^r$.

This is a geometric series with first term 4, common ratio 2 and 5 terms

$S = 4(2^5 - 1) = 124$

Answer: 124

5 Derivatives of tan, sec, cosec and cot (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Differentiate y = 5 cot 2x with respect to x,

and find the exact gradient when $x = \frac{\pi}{12}$.

Use the standard results for tan, sec, cosec and cot with the chain rule

$\frac{dy}{dx} = -10\ \text{cosec}^2 2x$

At $x = \frac{\pi}{12}$: $\frac{dy}{dx} = -40$

Answer: $\mathbf{\frac{dy}{dx} = -10\ \text{cosec}^2 2x}$; gradient $\mathbf{-40}$

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6 Rationalising the denominator (AQA, Edexcel, OCR, MEI, Eduqas)

Express $\frac{8 + 5\sqrt{2}}{\sqrt{2}}$ in the form $p + q\sqrt{2}$, where p and q are integers.

Multiply top and bottom by $\sqrt{2}$:

$\frac{(8 + 5\sqrt{2})\sqrt{2}}{2} = \frac{8\sqrt{2} + 10}{2}$

$= 5 + 4\sqrt{2}$

Answer: $5 + 4\sqrt{2}$

7 Scalar product and angle between vectors (CIE)

$\mathbf{a} = \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 3 \\ 3 \\ 0 \end{pmatrix}$.

Find the angle between a and b, to 1 d.p.

$\mathbf{a}\cdot\mathbf{b} = (2)(3) + (1)(3) + (-2)(0) = 9$

$|\mathbf{a}| = \sqrt{9}$, $|\mathbf{b}| = \sqrt{18}$

$\cos\theta = \frac{9}{\sqrt{9}\sqrt{18}} = 0.7071$

$\theta = 45.0°$

Answer: 45.0°

8 Rod suspended by two vertical strings (AQA, Edexcel, OCR, MEI, Eduqas)

A uniform rod AB of length 6 m and mass 6 kg hangs horizontally from two vertical strings attached 0.5 m from A and 0.5 m from B.

A particle of mass 2 kg is attached 2 m from A. Take $g = 9.8$ m s^{-2}. Find the tension in each string.

Moments about the string near A: $T_2(5) = 6\,g(2.5) + 2\,g(1.5)$

$T_2 = 35.28$

$T_1 + T_2 = 8\,g$ so $T_1 = 78.4 - 35.28$

Answer: Near A: 43.12 N; near B: 35.28 N

9 Exponential and power models (log-linear) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Data are modelled by $y = ab^x$, where a and b are constants.

A plot of $\log_{10} y$ against x is a straight line with intercept 0.477 and gradient 0.699.

Find a and b to 2 significant figures.

$\log_{10} y = \log_{10} a + x \log_{10} b$

$\log_{10} a = 0.477$, so $a = 10^{0.477} = 3.0$

$\log_{10} b = 0.699$, so $b = 10^{0.699} = 5.0$

Answer: a = 3.0, b = 5.0

10 Tree diagram (All boards)

The tree diagram shows probabilities for event E, followed by event F.

Find the probability that exactly one of E and F occurs.

P(E and not F) = 0.25 × 0.7 = 0.175

P(not E and F) = 0.75 × 0.2 = 0.15

Exactly one: 0.175 + 0.15 = 0.325

Answer: 0.325