Day 227 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Arithmetic sequences and series (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the sum of all the multiples of 8 that lie between 45 and 240.

First multiple = 8 × 6 = 48, last multiple = 8 × 29 = 232
This is an arithmetic series with d = 8 and 24 terms

$S = \frac{24}{2}(48 + 232) = 3360$

Answer: 3360

2 Arc length and sector area (radians) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A sector of a circle has radius 12 cm and arc length 21.6 cm.
Find (a) the angle of the sector in radians, (b) the area of the sector.

$s = r\theta$, so $\theta = \frac{21.6}{12} = 1.8$

$\text{Area} = \frac{1}{2}r^2\theta = \frac{1}{2} \times 12^2 \times 1.8 = 129.6$

Answer: (a) 1.8 rad (b) 129.6 cm^2

3 Will it move? Limiting friction (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A box of mass 5 kg rests on a rough horizontal floor, with coefficient of friction 0.6. A horizontal force of 24 N is applied to the box.

Take $g = 9.8$ m s^{-2}.
Determine whether the box moves. Find the friction force, or the acceleration if it moves.

$R = 5 \times 9.8 = 49$, maximum friction $\mu R = 0.6 \times 49 = 29.4$
$24 < 29.4$ so the box stays at rest
Friction balances the applied force: $F = 24$
Answer: Does not move; friction = 24 N

4 Large data set: outliers (AQA, Edexcel, OCR, MEI, Eduqas)

Large data set: mean temperature (°C) at Heathrow has mean 20, sd 4.
Outliers are more than 3 sd from the mean.
Are the values 31 and 9 outliers?

Limits: $20 \pm 3 \times 4$, so from 8 to 32
31 is inside the limits, so it is not an outlier
9 is inside the limits, so it is not an outlier
Answer: 31: not an outlier; 9: not an outlier

5 Double angle equations (All boards)

Solve $\sin 2x = 3 \sin x$ for $0 \leq x \leq 360°$,
giving answers to 1 d.p. where necessary.

Use the double angle formula: $2 \sin x \cos x - 3 \sin x = 0$
$\sin x(2 \cos x - 3) = 0$
$\sin x = 0$: $x = 0.0°, 180.0°, 360.0°$
$\cos x = \frac{3}{2}$ has no solutions
Answer: x = 0.0°, 180.0°, 360.0°

6 Simplifying surds (AQA, Edexcel, OCR, MEI, Eduqas)

Simplify $\sqrt{72} + \sqrt{2} - \sqrt{50}$, giving your answer in the form $k\sqrt{2}$.

$\sqrt{72} = 6\sqrt{2}$, $\sqrt{2} = \sqrt{2}$, $\sqrt{50} = 5\sqrt{2}$
$6\sqrt{2} + \sqrt{2} - 5\sqrt{2} = 2\sqrt{2}$
Answer: $2\sqrt{2}$

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7 Inclined plane: force to hold or move the particle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A block of mass 10 kg is held on a rough plane ($\tan\alpha = \frac{4}{3}$, $\mu = 0.5$) by a force P N up the plane, as shown.

Take $g = 9.8$ m s^{-2}. The block is about to move down the plane. Find P.

$R = mg\cos\alpha = 98 \times 3/5 = 58.8$
Friction (limiting) $F = \mu R = 29.4$
Moving down, friction acts up: $P + F = mg\sin\alpha$
$mg\sin\alpha = 78.4$
Answer: 49 N

8 R cos(θ − α) form: express and maximum (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Express $9\cos\theta + 12\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < 90°$. Give α to 1 d.p.

$R\cos\alpha = 9$, $R\sin\alpha = 12$
$R = \sqrt{9^2 + 12^2} = 15$
$\tan\alpha = \frac{12}{9}$, so $\alpha = 53.1°$
Answer: $15\cos(\theta - 53.1°)$

9 Parametric curve meeting a line (AQA, Edexcel, OCR, MEI, Eduqas)

A curve has parametric equations $x = t - 3$, $y = -t^2 + 2t - 1$.
The line $y = x - 4$ meets the curve at two points.
Find the coordinates of these points.

Substitute x and y into the line: $-t^2 + 2t - 1 = t - 7$
$-t^2 + t + 6 = 0$, so $-(t - 3)(t + 2) = 0$
$t = -2$ or $t = 3$, giving the points $(-5, -9)$ and $(0, -4)$
Answer: $(-5, -9)$ and $(0, -4)$

10 Finding mean and standard deviation from two probabilities (All boards)

The random variable X is normally distributed.
$P(X < 72.5) = 0.0668$ and $P(X > 85) = 0.1587$.
Find the mean and standard deviation (3 s.f.).

$z_1 = -1.500$ and $z_2 = 0.9998$
$72.5 = \mu + (-1.500)\sigma$ and $85 = \mu + 0.9998\sigma$
Subtract: $12.5 = 2.500\,\sigma$, so $\sigma = 5.00$
$\mu = 72.5 - (-1.500)(5.000) = 80.0$
Answer: $\mu = 80.0$, $\sigma = 5.00$