Day 226 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Unknown force from acceleration (vectors) (AQA, Edexcel, OCR, MEI, Eduqas)

A particle of mass 2 kg is acted on by forces $(-3\mathbf{i} + 7\mathbf{j})$ N and $\mathbf{P}$ N only. Its acceleration is $(-\mathbf{j})$ m s^{-2}.
Find $\mathbf{P}$, and its magnitude to 3 significant figures.

F = ma: $(-3\mathbf{i} + 7\mathbf{j}) + \mathbf{P} = 2(-\mathbf{j}) = (-2\mathbf{j})$
$\mathbf{P} = 3\mathbf{i} - 9\mathbf{j}$
$|\mathbf{P}| = \sqrt{(3^2 + -9^2)}$
Answer: $\mathbf{P} = (3\mathbf{i} - 9\mathbf{j})$ N; $|\mathbf{P}| = 9.49$ N

2 Completing the square (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Write $-x^2 - 6x - 21$ in the form $a(x + p)^2 + q$.
(b) Write down the coordinates of the maximum point of $y = -x^2 - 6x - 21$.

Take out the factor -1: $-x^2 - 6x - 21 = -(x^2 + 6x) - 21$
$= -[(x + 3)^2 - 9] - 21$
$= -(x + 3)^2 - 12$
$(x + 3)^2 \geq 0$ with equality at $x = -3$, so the maximum is -12 at $x = -3$.
Answer: (a) $-(x + 3)^2 - 12$ (b) $(-3, -12)$

3 3D vectors: magnitude, distance and unit vectors (All boards)

The vector $\mathbf{a} = \begin{pmatrix} -2 \\ 1 \\ -2 \end{pmatrix}$.
Find $|\mathbf{a}|$ and a unit vector in the direction of $\mathbf{a}$.

$|\mathbf{a}| = \sqrt{2^2 + 1^2 + 2^2} = \sqrt{9} = 3$
Unit vector $= \frac{1}{3}\mathbf{a}$
Answer: $|\mathbf{a}| = 3$, unit vector $\frac{1}{3}\begin{pmatrix} -2 \\ 1 \\ -2 \end{pmatrix}$

4 Quadratic inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $x^2 + 3x + 2 < 0$.

Factorise: $(x + 1)(x + 2) < 0$
Critical values: $x = -2$ and $x = -1$
Sketch the ∪-shaped parabola: $-2 < x < -1$
Answer: $-2 < x < -1$

5 Binomial critical region (All boards)

$X \sim B(50, p)$. H_0: $p = 0.2$ is to be tested against H_1: $p > 0.2$ at the 10% significance level.
Find the critical region and the actual significance level.

Under H_0, $X \sim B(50, 0.2)$
$P(X \geq 15) = 0.0607 \leq 0.1$
$P(X \geq 14) = 0.1106 > 0.1$
The critical region is $X \geq 15$
Answer: $X \geq 15$; actual significance level 6.07%

6 Triangle formed by two lines and an axis (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Line l_1 has equation $x - y = 2$ and line l_2 has equation $x - 2y = -1$.
The lines meet at P. Find the area of the triangle formed by l_1, l_2 and the x-axis.

Solve simultaneously: P = (5, 3)
l_1 meets y = 0 at x = 2, and l_2 meets y = 0 at $x = -1$
Base = 3, height = 3
Area = $\frac{1}{2} \times$ base $\times$ height = $\frac{9}{2}$

Answer: $\frac{9}{2}$

7 Equations using $\sin^2 x + \cos^2 x = 1$ (All boards)

Solve $6\sin^2 x - 7\cos x - 8 = 0$ for $0 \le x \le 360°$,
giving answers to 1 d.p. where necessary.

Use $\sin^2 x = 1 - \cos^2 x$: $-6\cos^2 x - 7\cos x - 2 = 0$, i.e. $6\cos^2 x + 7\cos x + 2 = 0$
$(2\cos x + 1)(3\cos x + 2) = 0$, so $\cos x = -\frac{1}{2}$ or $-\frac{2}{3}$
$\cos x = -\frac{1}{2}$: x = 120.0°, 240.0°
$\cos x = -\frac{2}{3}$: x = 131.8°, 228.2°

Answer: x = 120.0°, 131.8°, 228.2°, 240.0°

8 Differentiating position vectors (AQA, Edexcel, OCR, MEI, Eduqas)

A particle moves so that its position vector relative to O at time t s is $\mathbf{r} = (2t^2 + 4t)\mathbf{i} + (3t^3 - 3t)\mathbf{j}$ metres.
Find the velocity when t = 1, and the speed at that time.

$\mathbf{v} = d\mathbf{r}/dt = (4t + 4)\mathbf{i} + (9t^2 - 3)\mathbf{j}$
t = 1: $\mathbf{v} = (8)\mathbf{i} + (6)\mathbf{j}$
Speed = $\sqrt{(8^2 + 6^2)}$

Answer: $\mathbf{v} = (8)\mathbf{i} + (6)\mathbf{j}$ m s^{-1}; speed 10 m s^{-1}

9 Finding mean and standard deviation from two probabilities (All boards)

The masses (kg) of parcels are normally distributed.
P(X < 92) = 0.0228 and P(X > 104) = 0.1587.
Find the mean and standard deviation (3 s.f.).

$z_1 = -1.999$ and $z_2 = 0.9998$
$92 = \mu + (-1.999)\sigma$ and $104 = \mu + 0.9998\sigma$
Subtract: $12 = 2.999\,\sigma$, so $\sigma = 4.00$
$\mu = 92 - (-1.999)(4.001) = 100$

Answer: μ = 100, σ = 4.00

10 Exponential growth and decay models (All boards)

A car's value, £V, after t years is given by $V = 20000 \times 0.8^t$.
Find (a) the value after 3 years, (b) when it first falls below £4000, to 3 s.f.

(a) $V = 20000 \times 0.8^3 = 10240$
(b) $0.8^t < 0.2$, so $t \ln 0.8 < \ln 0.2$
$\ln 0.8 < 0$, so the inequality reverses: $t > \frac{\ln 0.2}{\ln 0.8} = 7.213$

Answer: (a) £10240 (b) t > 7.21 years