Day 225 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Speed and direction from a velocity vector (AQA, Edexcel, OCR, MEI, Eduqas)

A drone moves with constant velocity $(5\mathbf{i} + 12\mathbf{j})$ m s^{-1}, where $\mathbf{i}$ and $\mathbf{j}$ are unit vectors east and north.
Find its speed and the acute angle its direction makes with $\mathbf{i}$, to 1 d.p.

Speed = $\sqrt{(5^2 + 12^2)} = 13$
$\tan\theta = 12 \div 5$
Answer: 13 m s^{-1}; 67.4°

2 Expectation and variance of aX + b (AQA, OCR, MEI, Eduqas, CIE)

The random variable X has E(X) = 3 and Var(X) = 9.
Find (a) E(−2X + 5),
(b) Var(−2X + 5), (c) E(X^2).

(a) E(aX + b) = aE(X) + b = (−2) × 3 + 5 = −1
(b) Var(aX + b) = a^{2}Var(X) = 4 × 9 = 36
(c) Var(X) = E(X^2) − [E(X)]^2, so E(X^2) = 9 + 9 = 18
Answer: (a) −1 (b) 36 (c) 18

3 Proof using parity (AQA, Edexcel, OCR, MEI, Eduqas)

Prove that $n^3 + 3n + 13$ is odd for all integers n.

$n^3 + 3n = n(n^2 + 3)$
If n is even, $n(n^2 + 3)$ is even.
If n is odd, n^2 is odd and $n^2 + 3$ is even, so $n(n^2 + 3)$ is even.
So $n^3 + 3n$ is always even, and adding 13 gives an odd number.
Answer: expression is odd

4 Binomial two-tailed test (All boards)

The probability that a student passes a driving test first time is thought to be 0.25. In a sample of 25 students, 13 pass first time.
Test, at the 5% significance level, whether the probability is different from 0.25.

H_0: p = 0.25, H_1: p ≠ 0.25. Under H_0, X ~ B(25, 0.25)
P(X ≥ 13) = 1 − P(X ≤ 12) = 0.0034
Two-tailed: compare with 0.025. 0.0034 < 0.025, so reject H_0
There is sufficient evidence that the probability is different from 0.25
Answer: reject H_0: sufficient evidence that the probability is different from 0.25

5 Arithmetic sequences and series (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The first term of an arithmetic series is 4 and the common difference is 4.
The sum of the first n terms is 1200. Find the value of n.

$S_n = \frac{n}{2}(2 \times 4 + (n - 1) \times 4) = 1200$

$4n^2 + 4n - 2400 = 0$
$4(n - 24)(n + 25) = 0$
n must be positive, so n = 24
Answer: n = 24

6 Parametric differentiation: tangents (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has parametric equations $x = 2\sin t$, $y = 3\cos 2t$.

Find $\frac{dy}{dx}$ in terms of t, and its exact value when $t = \frac{\pi}{6}$.

$\frac{dx}{dt} = 2\cos t$, $\frac{dy}{dt} = -6\sin 2t$

$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = -\frac{3\sin 2t}{\cos t}$

At $t = \frac{\pi}{6}$: $\frac{dy}{dx} = -3$

Answer: $\frac{dy}{dx} = -\frac{3\sin 2t}{\cos t}$; value -3

7 Jumping to a different journey: two-stage motion (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A train accelerates uniformly from rest at 2 m s^{-2} for 8 s, then travels at constant speed for 15 s, and finally decelerates uniformly at 1 m s^{-2} to rest.

Find the total distance travelled and the total time of the journey.

Max speed = 2 × 8 = 16

Stage 1: ½ × 8 × 16 = 64; stage 2: 16 × 15 = 240

Stage 3: time = 16 ÷ 1 = 16, distance = ½ × 16 × 16 = 128

Answer: 432 m; 39 s

8 Stationary points of exponential and log curves (All boards)

Find the coordinates of the stationary point of the curve $y = xe^{-3x}$, and determine its nature.

$\frac{dy}{dx} = (1 - 3x)e^{-3x} = 0$

$x = \frac{1}{3}$

$\frac{d^2y}{dx^2} = 3(3x - 2)e^{-3x}$

At $x = \frac{1}{3}$: $\frac{d^2y}{dx^2} = -\frac{3}{e} < 0$, so a maximum

Answer: $\left(\frac{1}{3}, \frac{1}{3e}\right)$ maximum

9 Perpendicular bisector (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A is the point (−1, 2) and B is the point (4, 6).

Find the perpendicular bisector of AB as $ax + by + c = 0$, with integer a, b, c.

Midpoint of AB = $(\frac{3}{2}, 4)$

Gradient of AB = $\frac{4}{5}$

Perpendicular gradient = $-\frac{5}{4}$

$y - 4 = -\frac{5}{4}(x - \frac{3}{2})$, which gives $10x + 8y - 47 = 0$

Answer: $10x + 8y - 47 = 0$

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10 Exponential growth and decay models (All boards)

A car's value, £V, after t years is given by $V = 12000 \times 0.8^t$.
Find (a) the value after 3 years, (b) when it first falls below £2400, to 3 s.f.

(a) $V = 12000 \times 0.8^3 = 6144$

(b) $0.8^t < 0.2$, so $t \ln 0.8 < \ln 0.2$

$\ln 0.8 < 0$, so the inequality reverses: $t > \dfrac{\ln 0.2}{\ln 0.8} = 7.213$

Answer: (a) £6144 (b) t > 7.21 years