Day 224 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Vectors in a triangle (proof and ratio) (All boards)

In triangle OAB, OA = a and OB = b. M lies on AB with AM : MB = 3 : 2, and N lies on OB with ON : NB = 3 : 2. Find OM and NM in terms of a and b.

AB = b − a, so AM = $\frac{3}{5}$(b − a)

OM = a + AM = $\frac{2a}{5} + \frac{3b}{5}$

ON = $\frac{3b}{5}$, so NM = OM − ON = $\frac{2a}{5}$

Answer: OM = $\frac{2a}{5} + \frac{3b}{5}$, NM = $\frac{2a}{5}$

2 Expectation and variance of aX + b (AQA, OCR, MEI, Eduqas, CIE)

The random variable X has E(X) = 5 and Var(X) = 3. Find (a) E(2X + 2), (b) Var(2X + 2), (c) E(X^2).

(a) E(aX + b) = aE(X) + b = 2 × 5 + 2 = 12

(b) Var(aX + b) = a^2Var(X) = 4 × 3 = 12

(c) Var(X) = E(X^2) − [E(X)]^2, so E(X^2) = 3 + 25 = 28

Answer: (a) 12 (b) 12 (c) 28

3 Solving exponential equations using logarithms (All boards)

Solve 2^{2x+2} = 102, giving your answer to 3 s.f.

Take logs: (2x + 2) ln 2 = ln 102

2x + 2 = $\frac{\ln 102}{\ln 2}$ = 6.6724

x = 2.336

Answer: x = 2.34

4 Coefficients in a binomial expansion (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the term independent of x in the expansion of $(4x - \frac{3}{x})^4$.

The general term is C(4, k)$(4x)^{4-k}(-3/x)^k$, with power of x equal to 4 − 2k

Independent of x: 4 − 2k = 0, so k = 2

Term = 6 × 4^2 × $(-3)^2$ = 864

Answer: 864

5 Constant acceleration, two stages of working (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moving in a straight line with constant acceleration increases its speed from 15 m s^{-1} to 21 m s^{-1} while covering 108 m. Find (a) the acceleration, (b) the time taken.

(a) $v^2 = u^2 + 2as$: 441 = 225 + 216 a, so a = 1

(b) v = u + at: 21 = 15 + 1 t

t = 6 ÷ 1 = 6

Answer: (a) 1 m s^{-2} (b) 6 s

6 Discriminant problems (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The equation $x^2 + 2kx + (k + 2) = 0$ has two distinct real roots. Find the possible values of k.

$b^2 - 4ac = (2k)^2 - 4(k + 2) = 4k^2 - 4k - 8$

$= 4(k^2 - k - 2) = 4(k - 2)(k + 1)$

Need $b^2 - 4ac > 0$, so k < −1 or k > 2.

Answer: k < −1 or k > 2

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7 Constant acceleration with vectors (AQA, Edexcel, OCR, MEI, Eduqas)

A particle has velocity $(2\mathbf{i} - 4\mathbf{j})$ m s^{-1} at time t = 0 and moves with constant acceleration $(-2\mathbf{i} + 2\mathbf{j})$ m s^{-2}.
Find its velocity and speed when t = 5.

$\mathbf{v} = \mathbf{u} + \mathbf{a}t = (2\mathbf{i} - 4\mathbf{j}) + 5(-2\mathbf{i} + 2\mathbf{j})$
$= -8\mathbf{i} + 6\mathbf{j}$
Speed $= \sqrt{(-8^2 + 6^2)}$
Answer: $\mathbf{v} = (-8\mathbf{i} + 6\mathbf{j})$ m s^{-1}; speed 10 m s^{-1}

8 Binomial expansion (any power) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Expand $(1 - 4x)^{-2}$ in ascending powers of x up to and including the term in x^3.
(b) State the range of values of x for which the expansion is valid.

$(1 + ax)^n = 1 + n(ax) + \frac{n(n-1)}{2!}(ax)^2 + \frac{n(n-1)(n-2)}{3!}(ax)^3 + \ldots$
Here n = −2 and a = −4:
coefficients: x: 8, x^2: 48, x^3: 256
Valid when $|4x| < 1$, that is $|x| < \frac{1}{4}$
Answer: (a) $1 + 8x + 48x^2 + 256x^3 + \ldots$ (b) $|x| < \frac{1}{4}$

9 Intersection of two lines in 3D (CIE)

Line l_1: $\mathbf{r} = \begin{pmatrix} 0 \\ -1 \\ -3 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 1 \\ -2 \end{pmatrix}$.
Line l_2: $\mathbf{r} = \begin{pmatrix} -6 \\ 2 \\ -7 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ -2 \\ 3 \end{pmatrix}$.
Show that l_1 and l_2 intersect and find the point.

i: $0 + 0\lambda = -6 + 3\mu$
j: $-1 + 1\lambda = 2 + (-2)\mu$
Solving i and j: $\lambda = -1$, $\mu = 2$
k check: $-3 + (-2)(-1) = -1$ and $-7 + 3(2) = -1$, equal
Point: $\begin{pmatrix} 0 \\ -2 \\ -1 \end{pmatrix}$
Answer: $\begin{pmatrix} \mathbf{0} \\ \mathbf{-2} \\ \mathbf{-1} \end{pmatrix}$

10 Finding mean and standard deviation from two probabilities (All boards)

The scores on a test are normally distributed.
P(X < 50) = 0.1587 and P(X > 70) = 0.1587.
Find the mean and standard deviation (3 s.f.).

$z_1 = -0.9998$ and $z_2 = 0.9998$
$50 = \mu + (-0.9998)\sigma$ and $70 = \mu + 0.9998\sigma$
Subtract: $20 = 2.000\,\sigma$, so $\sigma = 10.0$
$\mu = 50 - (-0.9998)(10.00) = 60.0$
Answer: $\boldsymbol{\mu} = 60.0$, $\boldsymbol{\sigma} = 10.0$