Day 223 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Beam on two supports: reactions (AQA, Edexcel, OCR, MEI, Eduqas)

A uniform beam AB of length 5 m and mass 60 kg rests horizontally on supports at C and D, where AC = 1 m and BD = 1.5 m.

A particle of mass 5 kg is placed on the beam 2 m from B. Take g = 9.8 m s^{-2}.

Find the reactions at C and D.

Moments about C: $R_D \times 2.5 = 60\,g \times 1.5 + 5\,g \times 2$

$R_D = 980 \div 2.5$

Resolve vertically: $R_C + R_D = 65\,g$

Answer: R_C = 245 N; R_D = 392 N

2 Completing the square (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Write $-2x^2 + 24x - 80$ in the form $a(x + p)^2 + q$.

(b) Write down the coordinates of the maximum point of $y = -2x^2 + 24x - 80$.

Take out the factor -2: $-2x^2 + 24x - 80 = -2(x^2 - 12x) - 80$

$= -2[(x - 6)^2 - 36] - 80$

$= -2(x - 6)^2 - 8$

$(x - 6)^2 \geq 0$ with equality at $x = 6$, so the maximum is -8 at $x = 6$.

Answer: (a) $-2(x - 6)^2 - 8$ (b) $(6, -8)$

3 Fixed-point iteration (All boards)

(a) Show that $x^2 - x - 7 = 0$ can be written as $x = \sqrt{x + 7}$.

(b) Use $x_{n+1} = \sqrt{x_n + 7}$ with $x_0 = 3$ to find x_1, x_2, x_3 to 4 d.p.

(a) $x^2 = x + 7$, so $x = \sqrt{x + 7}$

(b) $x_1 = 3.1623$

$x_2 = 3.1878$

$x_3 = 3.1918$

Answer: x_1 = 3.1623, x_2 = 3.1878, x_3 = 3.1918

4 Comparing box plots (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The box plots show the test scores for two groups.

(a) Find the interquartile range of each group.

(b) Compare the two distributions.

(a) Year 12: 65 − 35 = 30. Year 13: 65 − 45 = 20

(b) Medians: 50 (Year 12) and 55 (Year 13), so Year 13 has the higher median

IQRs: Year 12 has the larger IQR, so its scores are more spread out

Answer: (a) 30 and 20 (b) Year 13 has the higher median; Year 12 is more spread out (larger IQR), Year 13 more consistent

5 Quotient rule (All boards)

Find the x-coordinate(s) of the stationary point(s) of the curve $y = \frac{x}{x^2 + 2}$.

$$\frac{dy}{dx} = \frac{2 - x^2}{(x^2 + 2)^2}$$

Stationary points where $\frac{dy}{dx} = 0$, i.e. the numerator is zero

Numerator: $2 - x^2 = 0$

$x = -\sqrt{2}$ or $x = \sqrt{2}$

Answer: $x = -\sqrt{2}$, $x = \sqrt{2}$

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6 Modulus equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $|4x - 2| = |-2x - 5|$.

Either $4x - 2 = -2x - 5$ or $4x - 2 = -(-2x - 5)$

$4x - 2 = -2x - 5$ gives $x = -\frac{1}{2}$

$4x - 2 = -(-2x - 5)$ gives $x = \frac{7}{2}$

Answer: $x = -\frac{1}{2}$ and $x = \frac{7}{2}$

7 Beam about to tilt (AQA, Edexcel, OCR, MEI, Eduqas)

A uniform plank AB of length 5 m and mass 50 kg rests horizontally on supports at C and D, where AC = 0.5 m and DB = 2 m.

A girl of mass 25 kg walks from D towards B. Find how far she is from D when the plank is about to tilt.

About to tilt about D: reaction at C is zero

Moments about D: $50\text{ g} \times (3 - 2.5) = 25\text{ g} \times x$

$50 \times 0.5 = 25\,x$

Answer: 1 m

8 Intersection of two lines in 3D (CIE)

Line l_1: $\mathbf{r} = \begin{pmatrix} -4 \\ 2 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}$.

Line l_2: $\mathbf{r} = \begin{pmatrix} -1 \\ -1 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ 3 \\ 0 \end{pmatrix}$.

Show that l_1 and l_2 intersect and find the point.

i: $-4 + 2\lambda = -1 + 3\mu$

j: $2 + 0\lambda = -1 + 3\mu$

Solving i and j: $\lambda = 3$, $\mu = 1$

k check: $2 + 0(3) = 2$ and $2 + 0(1) = 2$, equal

Point: $\begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix}$

Answer: $\begin{pmatrix} \mathbf{2} \\ \mathbf{2} \\ \mathbf{2} \end{pmatrix}$

9 Hypothesis test for a normal mean (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Bottle volumes are normal, s.d. 4 ml. Set mean 50 ml; 9 bottles have mean 51.9 ml.

Test at the 5% level whether the true mean is greater than 50.

H_0: $\mu = 50$, H_1: $\mu > 50$. Under H_0, $\overline{X} \sim N(50, \frac{4^2}{9})$

$z = \frac{51.9 - 50}{4/\sqrt{9}} = 1.425$

$P(Z > 1.425) = 0.0771 > 0.05$

Do not reject H_0: there is insufficient evidence that the mean has increased

Answer: do not reject H_0: insufficient evidence of an increase in the mean

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10 Perpendicular lines and shortest distance (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line l has equation $y = -\frac{1}{2}x + 5$. The point P is $(-1, 8)$.

(a) Find the line through P perpendicular to l, as $ax + by + c = 0$ with integer a, b, c.
(b) Find the foot F and the exact distance PF.

(a) Gradient of l $= -\frac{1}{2}$, so the perpendicular has gradient 2

$y - 8 = 2(x + 1)$, giving $2x - y + 10 = 0$

(b) Solve $y = -\frac{1}{2}x + 5$ with $2x - y + 10 = 0$: F $= (-2, 6)$

$PF^2 = 1^2 + 2^2 = 5$

Answer: (a) $2x - y + 10 = 0$ (b) F$(-2, 6)$, PF $= \sqrt{5}$