Day 222 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Rates of change (connected rates) (All boards)

The radius of a circular oil slick increases at 0.2 cm per second.
Find the rate at which its area is increasing when r = 12 cm, in terms of π.

$A = \pi r^2$, so $\frac{dA}{dr} = 2\pi r$

$\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt} = 2\pi r \times 0.2$

When r = 12: $\frac{dA}{dt} = \frac{24\pi}{5}$

Answer: $\frac{24\pi}{5}$ **cm²/s**

2 Parametric to Cartesian (AQA, Edexcel, OCR, MEI, Eduqas)

A curve has parametric equations $x = 5t$, $y = \frac{3}{t}$, $t \neq 0$.

Find a Cartesian equation of the curve.

Multiply the two equations: $xy = 5t \times \frac{3}{t} = 15$

So $y = \frac{15}{x}$

Answer: $y = \frac{15}{x}$

3 Integrating velocity (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves in a straight line. At time t seconds its velocity is $v = 4t^2 + 6t + 2$ m s^{-1}.
Find the distance moved between t = 2 and t = 5. (The particle does not change direction.)

$s = \int_2^5 (4t^2 + 6t + 2)\, dt$

$= [\frac{4}{3}t^3 + 3t^2 + 2t]$ from t = 2 to t = 5

= 755/3 − 80/3

Answer: 225 m

4 R cos(θ − α) form: solving (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Given that $15\cos\theta + 8\sin\theta \equiv 17\cos(\theta - 28.1°)$,
solve $15\cos\theta + 8\sin\theta = -4$ for $0 \leq \theta \leq 360°$. (1 d.p.)

$17\cos(\theta - 28.1°) = -4$, so $\cos(\theta - 28.1°) = \frac{-4}{17}$

$\theta - 28.1° = \pm 103.6°$

$\theta = 28.1° + 103.6° = 131.7°$

$\theta = 28.1° - 103.6° = -75.5°$, so $\theta = 284.5°$

Answer: θ **= 131.7°, 284.5°**

5 Using a binomial expansion to estimate (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Expand $(1 + 5x)^{\frac{1}{2}}$ up to the x^2 term.
(b) Use x = 0.03 to estimate $\sqrt{1.15}$ to 5 decimal places.

(a) $(1 + ax)^n = 1 + n(ax) + \frac{n(n-1)}{2}(ax)^2 + \ldots$ with $n = \frac{1}{2}$, a = 5:

$= 1 + \frac{5}{2}x - \frac{25}{8}x^2$

(b) 1 + 5 × 0.03 = 1.15, so substitute x = 0.03:

$1 + \frac{5}{2} \times 0.03 - \frac{25}{8} \times 0.03^2 = 1.07219$

Answer: (a) $1 + \frac{5}{2}x - \frac{25}{8}x^2 + \ldots$ **(b) 1.07219**

6 Pulling at an angle: constant speed on rough surface (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A crate of mass 10 kg is dragged at constant speed along rough horizontal ground by a rope inclined at angle α above the horizontal, where $\cos\alpha = \frac{12}{13}$.

The coefficient of friction is 0.2. Take $g = 9.8$ m s^{-2}. Find the tension in the rope.

Vertical: $R + T\sin\alpha = mg$, so $R = 98 - \frac{5}{13}T$

Horizontal: $T\cos\alpha = \mu R$: $\frac{12}{13}T = 0.2(98 - \frac{5}{13}T)$

$T(\cos\alpha + \mu\sin\alpha) = \mu mg$: $T \times 1 = 0.2 \times 10 \times 9.8$ (since $\cos\alpha + \mu\sin\alpha = 1$)

Answer: 19.6 N

7 Proof by exhaustion (AQA, Edexcel, OCR, MEI, Eduqas)

Prove, by exhaustion, that a square number cannot be written in the form $8k + 3$, where k is an integer.

Write the integer as $n = 8q + j$, where $j = 0, 1, ..., 7$.

n^2 leaves the same remainder as j^2 when divided by 8.

$j = 0, 1, 2, 3, 4, 5, 6, 7$ give remainders 0, 1, 4, 1, 0, 1, 4, 1.

The only possible remainders are 0, 1, 4, never 3.

Answer: possible remainders 0, 1, 4, never 3

8 Regression line (AQA, Edexcel, OCR, MEI, Eduqas)

For 12 pairs of values, $\bar{x} = 8$, $\bar{y} = 40$, $S_{xx} = 100$ and $S_{xy} = 50$.

(a) Find the regression line of y on x in the form $y = a + bx$.

(b) Estimate y when $x = 10$.

(a) $b = \frac{S_{xy}}{S_{xx}} = \frac{50}{100} = 0.5$

$a = \bar{y} - b\bar{x} = 40 - 0.5 \times 8 = 36$

$y = 36 + 0.5x$

(b) $y = 36 + 0.5 \times 10 = 41$

Answer: y = 36 + 0.5x; y = 41

9 Newton-Raphson method (AQA, Edexcel, OCR, MEI, Eduqas)

$f(x) = x^3 + 3x - 3$. Use the Newton-Raphson method with $x_0 = 1$ to find x_1 and x_2, to 4 d.p.

$f'(x) = 3x^2 + 3$

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$x_1 = 1 - \frac{1.000}{6.000} = 0.8333$

$x_2 = 0.8333 - \frac{0.07870}{5.083} = 0.8179$

Answer: $x_1 = 0.8333$, $x_2 = 0.8179$

10 Binomial expected frequency (All boards)

A fair die is rolled 12 times and the number of sixes is X.

This is done 2000 times. Estimate the number of times that X = 5.

$X \sim B(12, \frac{1}{6})$

$P(X = 5) = {}^{12}C_5 \times (\frac{1}{6})^5 \times (\frac{5}{6})^7 = 0.02842$

Expected $= 2000 \times 0.02842 = 56.85$

Answer: about 57 times