Day 221 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Completing the square (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Write $x^2 + 4x - 7$ in the form $a(x + p)^2 + q$.
(b) Write down the coordinates of the minimum point of $y = x^2 + 4x - 7$.

$x^2 + 4x = (x + 2)^2 - 4$
So $x^2 + 4x - 7 = (x + 2)^2 - 11$
$(x + 2)^2 \geq 0$ with equality at $x = -2$, so the minimum is -11 at $x = -2$.
Answer: (a) $(x + 2)^2 - 11$ (b) $(-2, -11)$

2 Overtaking problem (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A motorcycle starts from rest and accelerates at 0.5 m s^{-2}. At that moment it is passed by a lorry travelling at constant speed 20 m s^{-1}.
Find how long it takes to catch up, and the distance travelled.

Same distance: $\frac{1}{2} \times 0.5 \times t^2 = 20\ t$
$t(0.25\ t - 20) = 0$, $t \neq 0$ so $t = 20 \div 0.25$
Distance $= 20 \times 80$
Answer: 80 s, 1600 m

3 3D vectors: magnitude, distance and unit vectors (All boards)

Points A and B have position vectors $\begin{pmatrix} -3 \\ 1 \\ -4 \end{pmatrix}$ and $\begin{pmatrix} -9 \\ -5 \\ 3 \end{pmatrix}$.

Find AB and the distance AB.

$\text{AB} = \text{OB} - \text{OA} = \begin{pmatrix} -6 \\ -6 \\ 7 \end{pmatrix}$

$|\text{AB}| = \sqrt{6^2 + 6^2 + 7^2} = \sqrt{121} = 11$

Answer: AB $= \begin{pmatrix} -6 \\ -6 \\ 7 \end{pmatrix}$, distance 11

4 Probability without replacement (All boards)

A bag has 6 green and 3 yellow beads; two are taken without replacement.
(a) Find P(exactly one green).
(b) Find P(at least one green).

(a) P(green then yellow) + P(yellow then green) $= 2 \times \frac{6}{9} \times \frac{3}{8} = \frac{1}{2}$

(b) $1 - \text{P(both yellow)} = 1 - \frac{3}{9} \times \frac{2}{8} = 1 - \frac{1}{12} = \frac{11}{12}$

Answer: (a) $\frac{1}{2}$ (b) $\frac{11}{12}$

5 Transformations from a graph (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the curve y = f(x). A and B are the turning points. Find the turning points of:
(a) y = f(x − 1) (b) y = 2f(x)

(a) y = f(x − 1): A(−2, 1) maps to (−1, 1), B(2, −2) maps to (3, −2)
(b) y = 2f(x): A(−2, 1) maps to (−2, 2), B(2, −2) maps to (2, −4)
Answer: (a) (−1, 1), (3, −2) (b) (−2, 2), (2, −4)

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6 Rough inclined plane: sliding down (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A block of mass 3 kg is released from rest on a rough plane inclined at angle α to the horizontal, where $\tan\alpha = \frac{3}{4}$.

The coefficient of friction is 0.125. Take $g = 9.8$ m s^{-2}. Find the acceleration down the plane, and the distance moved in 3 s.

$\cos\alpha = 4/5$, $\sin\alpha = 3/5$

$R = mg\cos\alpha$, $F = \mu R$

$ma = mg\sin\alpha - \mu mg\cos\alpha$, so $a = 9.8(3/5 - 0.125 \times 4/5)$

$s = \frac{1}{2}at^2 = \frac{1}{2} \times 4.9 \times 3^2$

Answer: 4.9 m s^{-2}; 22.05 m

7 Segment of a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The shaded region is the segment cut off by the chord AB of a circle with centre O and radius 8 cm. Angle AOB = 1.2 radians.

Find the area of the segment, to 3 s.f.

Sector area $= \frac{1}{2} \times 8^2 \times 1.2 = 38.4$

Triangle area $= \frac{1}{2} \times 8^2 \times \sin 1.2 = 29.83$

Segment = sector − triangle = 8.575 cm^2

Answer: 8.57 cm^2

8 Factor theorem with an unknown (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = x^3 - 2x^2 - 5x + k$.

Given that $(x + 2)$ is a factor of $f(x)$,

(a) find the value of k,

(b) factorise $f(x)$ completely.

$f(-2) = 0$: $-8 - 8 + 10 + k = 0$

$k = 6$

$f(x) = (x + 2)(x^2 - 4x + 3)$

$= (x - 3)(x - 1)(x + 2)$

Answer: (a) $k = 6$ (b) $f(x) = (x - 3)(x - 1)(x + 2)$

9 Finding mean and standard deviation from two probabilities (All boards)

The scores on a test are normally distributed.

$P(X < 45) = 0.1587$ and $P(X > 55) = 0.1587$.

Find the mean and standard deviation (3 s.f.).

$z_1 = -0.9998$ and $z_2 = 0.9998$

$45 = \mu + (-0.9998)\sigma$ and $55 = \mu + 0.9998\sigma$

Subtract: $10 = 2.000\ \sigma$, so $\sigma = 5.00$

$\mu = 45 - (-0.9998)(5.001) = 50.0$

Answer: $\mu = 50.0$, $\sigma = 5.00$

10 Equations with sec, cosec and cot (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $\tan^2 x - 3 = 0$ for $0 \le x \le 360°$,

giving answers to 1 d.p.

Use $\tan^2 x = \sec^2 x - 1$: $\sec^2 x - 4 = 0$

$(\sec x + 2)(\sec x - 2) = 0$, so $\sec x = -2$ or 2

$\sec x = -2$, so $\cos x = -\frac{1}{2}$: $x = 120.0°, 240.0°$

$\sec x = 2$, so $\cos x = \frac{1}{2}$: $x = 60.0°, 300.0°$

Answer: $x = 60.0°, 120.0°, 240.0°, 300.0°$