Day 220 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Driving force and resistance (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A van of mass 1000 kg moves along a straight horizontal road. The engine produces a driving force of 2000 N and the resistance to motion is 900 N.
Find the acceleration of the van.

Resultant force = 2000 − 900 = 1100 N
F = ma: 1100 = 1000 a
Answer: 1.1 m s^{-2}

2 Chain rule (All boards)

Given that $y = e^{3x-6}$,
find $\frac{dy}{dx}$, and the gradient of the curve when x = 2.

Chain rule: differentiate the outer function, then multiply by the derivative of the inner function
$\frac{dy}{dx} = 3e^{3x-6}$
When x = 2: gradient = 3
Answer: $\frac{dy}{dx} = 3e^{3x-6}$; gradient 3

3 Graph of the modulus of a quadratic (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the curve y = f(x), a quadratic.
(a) Find the coordinates of the local maximum point of y = |f(x)|.
(b) How many solutions has |f(x)| = 18?

(a) y = |f(x)| reflects the part of the curve below the x-axis in the x-axis:
(1, −16) becomes (1, 16)
(b) The maximum of the middle part is 16. Compare k = 18 with 16:
k > 16 gives 2 intersections with y = k.
Answer: (a) (1, 16) (b) 2

4 Mean and standard deviation from sums (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A summary of the ages, a years, of 7 members of a club gives $\Sigma a = 195$ and $\Sigma a^2 = 6131$.
Find the mean and the standard deviation (3 s.f.).

Mean = 195 ÷ 7 = 27.9
Variance = $\frac{6131}{7} - (27.857)^2 = 99.84$
Standard deviation = $\sqrt{99.84} = 9.99$
Answer: mean 27.9, standard deviation 9.99

5 Instantaneous rest and displacement (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle P moves on a straight line from O. At time t s its velocity is $6t^2 - 48t + 72$ m s^{-1}.
Find the first time P is at instantaneous rest, and its distance from O then.

v = 0: 6(t − 2)(t − 6) = 0, so t = 2 or t = 6
First time t = 2
$s = \int_0^2 v\,dt = [\,2t^3 - 24t^2 + 72t\,]$ from 0 to 2
Answer: t = 2 s; 64 m

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6 Sigma notation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Given that $\sum_{r=1}^{n}(3r + 4) = 325$, find the value of n.

Sum = $\frac{n}{2}(7 + 3n + 4) = 325$

$3n^2 + 11n - 650 = 0$
$(n - 13)(3n + 50) = 0$
n must be positive, so n = 13
Answer: n = 13

7 Equations with sec, cosec and cot (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $3\tan^2 x + \sec x - 1 = 0$ for $0 \leq x \leq 360°$,
giving answers to 1 d.p.

Use $\tan^2 x = \sec^2 x - 1$: $3\sec^2 x + \sec x - 4 = 0$

$(3 \sec x + 4)(\sec x - 1) = 0$, so $\sec x = -\frac{4}{3}$ or 1

$\sec x = -\frac{4}{3}$, so $\cos x = -\frac{3}{4}$: x = 138.6°, 221.4°

sec x = 1, so cos x = 1: x =
Answer: x = 138.6°, 221.4°

8 Partial fractions (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Express $\frac{-2x^2 - 3x + 3}{(x + 3)(x + 2)^2}$ in partial fractions.

$\frac{-2x^2 - 3x + 3}{(x + 3)(x + 2)^2} \equiv \frac{A}{(x + 3)} + \frac{B}{(x + 2)} + \frac{C}{(x + 2)^2}$
$-2x^2 - 3x + 3 \equiv A(x + 2)^2 + B(x + 3)(x + 2) + C(x + 3)$
x = −3 gives A = −6; x = −2 gives C = 1
Coefficients of x^2: −2 = A + B, so B = 4
Answer: $-\frac{6}{(x + 3)} + \frac{4}{(x + 2)} + \frac{1}{(x + 2)^2}$

9 Finding mean and standard deviation from two probabilities (All boards)

The scores on a test are normally distributed.
P(X < 68) = 0.0668 and P(X > 88) = 0.1587.
Find the mean and standard deviation (3 s.f.).

$z_1 = -1.500$ and $z_2 = 0.9998$
$68 = \mu + (-1.500)\sigma$ and $88 = \mu + 0.9998\sigma$
Subtract: $20 = 2.500\,\sigma$, so $\sigma = 8.00$
$\mu = 68 - (-1.500)(8.000) = 80.0$
Answer: μ = 80.0, σ = 8.00

10 Compound angle formulae (exact values) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Given that $\sin A = \frac{12}{13}$ and $\sin B = \frac{15}{17}$, where A is obtuse and B is acute,
find the exact value of cos(A − B).

$\cos A = -\frac{5}{13}$ and $\cos B = \frac{8}{17}$
$\cos(A - B) = \cos A \cos B + \sin A \sin B$
$= (-\frac{5}{13}) \times \frac{8}{17} + \frac{12}{13} \times \frac{15}{17} = \frac{140}{221}$
Answer: $\frac{140}{221}$