Day 22 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Basic trig equations (degrees) (All boards)

Solve $\cos x = \frac{1}{2}$ for $0 \leq x \leq 360°$.

Principal value $x = 60°$
Second solution: $360° - 60° = 300°$
Answer: x = 60°, 300°

2 Frequency table: mean and standard deviation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The table shows the number of siblings, x, of 51 students.
Calculate the mean and standard deviation (3 s.f.).

$\Sigma f = 51$, $\Sigma fx = 187$, $\Sigma fx^2 = 853$

Mean $= \frac{187}{51} = 3.67$

Variance $= \frac{853}{51} - (3.6667)^2 = 3.281$

Standard deviation = 1.81
Answer: mean 3.67, standard deviation 1.81

3 Proof using parity (AQA, Edexcel, OCR, MEI, Eduqas)

Prove that $n^2 + n + 6$ is even for all integers n.

$n^2 + n = n(n + 1)$
As 1 is odd, n and n + 1 have opposite parity,
so one of them is even and the product n(n + 1) is even.
Even + 6 = even, so $n^2 + n + 6$ is even.
Answer: n(n + 1) is even, so the expression is even

4 Poisson distribution (CIE)

X ~ Po(4), modelling the number of calls to a helpline per minute.
Find, to 3 significant figures,
(a) $P(X \leq 4)$
(b) $P(X \geq 4)$

(a) $P(X \leq 4) = P(X = 0) + ... + P(X = 4) = 0.629$
(b) $P(X \geq 4) = 1 - P(X \leq 3) = 0.567$
Answer: (a) 0.629 (b) 0.567

5 Particle in equilibrium: string and horizontal force (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle of mass 6 kg hangs in equilibrium on a light string inclined at α to the vertical, held by a horizontal force P N, as shown. $\tan \alpha = \frac{3}{4}$.

Take $g = 9.8$ m s^{-2}.
Find the tension in the string and the value of P.

$\cos \alpha = 4/5$, $\sin \alpha = 3/5$
Vertical: $T \cos \alpha = mg = 58.8$, so $T = 58.8 \times 5/4$
Horizontal: $P = T \sin \alpha = 73.5 \times 3/5$
Answer: T = 73.5 N; P = 44.1 N

6 Factor theorem with an unknown (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = 2x^3 + kx^2 - 2x - 8$.
Given that $(x + 1)$ is a factor of $f(x)$,
(a) find the value of k,
(b) factorise $f(x)$ completely.

$f(-1) = 0$: $-2 + k + 2 - 8 = 0$
$k = 8$
$f(x) = (x + 1)(2x^2 + 6x - 8)$
$= 2(x - 1)(x + 1)(x + 4)$
Answer: (a) k = 8 (b) f(x) = 2(x − 1)(x + 1)(x + 4)

7 R cos(θ − α) form: solving (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Given that $7\cos\theta + 24\sin\theta \equiv 25\cos(\theta - 73.7°)$,
solve $7\cos\theta + 24\sin\theta = -1$ for $0 \leq \theta \leq 360°$. (1 d.p.)

$25\cos(\theta - 73.7°) = -1$, so $\cos(\theta - 73.7°) = \frac{-1}{25}$
$\theta - 73.7° = \pm 92.3°$
$\theta = 73.7° + 92.3° = 166.0°$
$\theta = 73.7° - 92.3° = -18.6°$, so $\theta = 341.4°$
Answer: θ = 166.0°, 341.4°

8 Pulling at an angle: constant speed on rough surface (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A suitcase of mass 12 kg is dragged at constant speed along rough horizontal ground by a rope inclined at angle α above the horizontal, where $\cos\alpha = \frac{12}{13}$.

The coefficient of friction is 0.2. Take $g = 9.8$ m s^{-2}. Find the tension in the rope.

Vertical: $R + T\sin\alpha = mg$, so $R = 117.6 - \frac{5}{13}T$
Horizontal: $T\cos\alpha = \mu R$: $\frac{12}{13}T = 0.2(117.6 - \frac{5}{13}T)$
$T(\cos\alpha + \mu\sin\alpha) = \mu mg$: $T \times 1 = 0.2 \times 12 \times 9.8$ (since $\cos\alpha + \mu\sin\alpha = 1$)
Answer: 23.52 N

9 Differential equations in context (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A cup of tea cools in a room at 20°C: $\frac{dT}{dt} = -k(T - 20)$. Initially $T = 80$ and after 5 minutes $T = 60$.
Find k to 3 s.f. and the time to cool to 30°C, to 3 s.f.

$\frac{1}{T - 20}\,dT = -k\,dt$, so $\ln(T - 20) = -kt + c$, i.e. $T = 20 + 60e^{-kt}$
$t = 5$: $60 = 20 + 60e^{-5k}$, so $e^{-5k} = \frac{2}{3}$
$k = \frac{1}{5}\ln\frac{60}{40} = 0.08109$
$30 = 20 + 60e^{-kt}$: $t = \frac{1}{k}\ln\frac{60}{10} = 22.10$
Answer: k = 0.0811, t = 22.1 minutes

10 Rationalising the denominator (AQA, Edexcel, OCR, MEI, Eduqas)

Express $\frac{3 - 3\sqrt{3}}{\sqrt{3}}$ in the form $p + q\sqrt{3}$, where p and q are integers.

Multiply top and bottom by $\sqrt{3}$:
$\frac{(3 - 3\sqrt{3})\sqrt{3}}{3} = \frac{3\sqrt{3} - 9}{3}$
$= -3 + \sqrt{3}$
Answer: $-3 + \sqrt{3}$