Day 219 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Stratified sampling (AQA, Edexcel, OCR, MEI, Eduqas)

A school has 32 students in Year 7, 60 in Year 8 and 32 in Year 9.
A stratified sample of size 31 is taken. Find the number selected from each group.

Total = 32 + 60 + 32 = 124

Sampling fraction = 31/124 = $\frac{1}{4}$

Year 7: 32 ÷ 4 = 8
Year 8: 60 ÷ 4 = 15
Year 9: 32 ÷ 4 = 8
Answer: Year 7 8, Year 8 15, Year 9 8

2 Proof by exhaustion (AQA, Edexcel, OCR, MEI, Eduqas)

Prove, by exhaustion, that a cube number cannot be written in the form 9k + 2, where k is an integer.

Write the integer as n = 9q + j, where j = 0, 1, ..., 8.
n^3 leaves the same remainder as j^3 when divided by 9.
j = 0, 1, 2, 3, 4, 5, 6, 7, 8 give remainders 0, 1, 8, 0, 1, 8, 0, 1, 8.
The only possible remainders are 0, 1, 8, never 2.
Answer: possible remainders 0, 1, 8, never 2

3 Double angle equations (All boards)

Solve sin 2x + cos x = 0 for 0 ≤ x ≤ 360°,
giving answers to 1 d.p. where necessary.

Use the double angle formula: 2 sin x cos x + cos x = 0
cos x(2 sin x + 1) = 0
cos x = 0: x = 90.0°, 270.0°
sin x = $-\frac{1}{2}$: x = 210.0°, 330.0°
Answer: x = 90.0°, 210.0°, 270.0°, 330.0°

4 Simplifying algebraic indices (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Simplify $\frac{(3x^2)^3 \times 3x^2}{x^2}$.

$(3x^2)^3 = 27x^6$
Numerator = $27 \times 3x^8 = 81x^8$
Divide: $\frac{81}{1} = 81$ and $x^8 \div x^2 = x^6$
Answer: $81x^6$

5 Impulse of a constant force: v = u + at with forces (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A trolley of mass 8 kg is moving on a smooth horizontal surface at 7 m s^{-1} when a constant horizontal force of 12 N, in the direction of motion, acts for 3 s.
Find the speed of the trolley at the end of this time and the distance it moves.

F = ma: a = 12 ÷ 8 = 1.5
v = u + at = 7 + 1.5 × 3
s = ut + ½at^2
Answer: 11.5 m s^{-1}; 27.75 m

6 Combined inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the set of values of x for which
$4x - 1 \leq 3$ and $x^2 + 8x + 12 \geq 0$.

$4x - 1 \leq 3$ gives $x \leq 1$
$x^2 + 8x + 12 \geq 0$: $(x + 6)(x + 2) \geq 0$ gives $x \leq -6$ or $x \geq -2$
Both must hold: $x \leq -6$ or $-2 \leq x \leq 1$
Answer: $x \leq -6$ or $-2 \leq x \leq 1$

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7 Differentiating position vectors (AQA, Edexcel, OCR, MEI, Eduqas)

A model boat moves so that its position vector relative to O at time t s is r = $(3t^2)\mathbf{i} + (t^3 - 3t)\mathbf{j}$ metres.
Find the velocity when t = 2, and the speed at that time.

v = dr/dt = $(6t)\mathbf{i} + (3t^2 - 3)\mathbf{j}$
t = 2: v = $(12)\mathbf{i} + (9)\mathbf{j}$
Speed = $\sqrt{(12^2 + 9^2)}$
Answer: v = (12)i + (9)j m s^{-1}; speed 15 m s^{-1}

8 Finding mean and standard deviation from two probabilities (All boards)

The random variable X is normally distributed.
P(X < 45) = 0.1587 and P(X > 57.5) = 0.0668.
Find the mean and standard deviation (3 s.f.).

$z_1 = -0.9998$ and $z_2 = 1.500$
$45 = \mu + (-0.9998)\sigma$ and $57.5 = \mu + 1.500\sigma$
Subtract: $12.5 = 2.500\ \sigma$, so $\sigma = 5.00$
$\mu = 45 - (-0.9998)(5.000) = 50.0$
Answer: μ = 50.0, σ = 5.00

9 Small angle approximations (AQA, Edexcel, OCR, MEI, Eduqas)

For small θ (in radians), find an approximation of the
form $1 + p\theta + q\theta^2$ for
$\sin 5\theta + \cos 3\theta + 3\tan\theta$.

$\sin 5\theta \approx 5\theta$, $\cos 3\theta \approx 1 - \frac{9\theta^2}{2}$, $\tan\theta \approx \theta$

Sum $\approx 1 + 5\theta - \frac{9\theta^2}{2} + 3\theta$

$\approx 1 + 8\theta - \frac{9\theta^2}{2}$

Answer: p = 8, q = $-\frac{9}{2}$

10 Compound angle formulae (exact values) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Given that $\sin A = \frac{3}{5}$ and $\cos B = \frac{8}{17}$, where A and B are acute,
find the exact value of sin(A + B).

$\cos A = \frac{4}{5}$ and $\sin B = \frac{15}{17}$
sin(A + B) = sin A cos B + cos A sin B
$= \frac{3}{5} \times \frac{8}{17} + \frac{4}{5} \times \frac{15}{17} = \frac{84}{85}$
Answer: $\frac{84}{85}$