Day 218 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Simplifying algebraic indices (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Simplify $\frac{(2x^3)^2 \times 3x^3}{x^8}$.

$(2x^3)^2 = 4x^6$

Numerator $= 4 \times 3x^9 = 12x^9$

Divide: $\frac{12}{1} = 12$ and $x^9 \div x^8 = x$

Answer: $12x$

2 Will it move? Limiting friction (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A box of mass 15 kg rests on a rough horizontal floor, with coefficient of friction 0.2. A horizontal force of 35 N is applied to the box.

Take $g = 9.8$ m s^{-2}.

Determine whether the box moves. Find the friction force, or the acceleration if it moves.

$R = 15 \times 9.8 = 147$, maximum friction $\mu R = 0.2 \times 147 = 29.4$

$35 > 29.4$ so the box moves, $F = 29.4$

$35 - 29.4 = 15\,a$ (3 s.f.)

Answer: It moves; $a = 0.373$ m s^{-2}

3 Quadratic inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $-4x^2 - 10x + 6 > 0$.

Factorise: $-2(x + 3)(2x - 1) > 0$

Critical values: $x = -3$ and $x = \frac{1}{2}$

Sketch the $\cap$-shaped parabola: $-3 < x < \frac{1}{2}$

Answer: $-3 < x < \frac{1}{2}$

4 Probability without replacement (All boards)

A bag has 4 red and 8 blue counters; two are taken without replacement.

(a) Find P(exactly one red).

(b) Find P(at least one red).

(a) P(red then blue) + P(blue then red) $= 2 \times \frac{4}{12} \times \frac{8}{11} = \frac{16}{33}$

(b) $1 - $ P(both blue) $= 1 - \frac{8}{12} \times \frac{7}{11} = 1 - \frac{14}{33} = \frac{19}{33}$

Answer: (a) $\frac{16}{33}$ (b) $\frac{19}{33}$

5 Product rule (All boards)

Given that $y = (x + 3)e^{-3x}$,

find $\frac{dy}{dx}$, and its value when $x = 0$.

Product rule with $u = x + 3$, $v = e^{-3x}$

$u' = 1$, $v' = -3e^{-3x}$

$\frac{dy}{dx} = u'v + uv' = (-3x - 8)e^{-3x}$

When $x = 0$: $\frac{dy}{dx} = -8$

Answer: $\frac{dy}{dx} = (-3x - 8)e^{-3x}$; value -8

6 Transformations from a graph (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the curve y = f(x). A and B are the turning points. Find the turning points of:
(a) y = f(−x) (b) y = f(x) − 3

(a) y = f(−x): A(−3, 3) maps to (3, 3), B(1, −5) maps to (−1, −5)
(b) y = f(x) − 3: A(−3, 3) maps to (−3, 0), B(1, −5) maps to (1, −8)
Answer: (a) (3, 3), (−1, −5) (b) (−3, 0), (1, −8)

7 Pulling at an angle: constant speed on rough surface (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A suitcase of mass 12 kg is dragged at constant speed along rough horizontal ground by a rope inclined at angle α above the horizontal, where $\cos\alpha = \frac{3}{5}$.

The coefficient of friction is 0.5. Take g = 9.8 m s^{-2}. Find the tension in the rope.

Vertical: $R + T\sin\alpha = mg$, so $R = 117.6 - \frac{4}{5}T$

Horizontal: $T\cos\alpha = \mu R$: $\frac{3}{5}T = 0.5(117.6 - \frac{4}{5}T)$

$T(\cos\alpha + \mu\sin\alpha) = \mu mg$: $T \times 1 = 0.5 \times 12 \times 9.8$ (since $\cos\alpha + \mu\sin\alpha = 1$)
Answer: 58.8 N

8 Binomial: smallest n (All boards)

Given that each component fails independently with probability 0.4, find the least number of components needed for the probability of at least one success to exceed 0.9.

P(at least one) = $1 - 0.6^n > 0.9$
$0.6^n < 0.1$
$n > \frac{\log 0.1}{\log 0.6} = 4.508$
Answer: n = 5

9 3D vectors: magnitude, distance and unit vectors (All boards)

The vector $\mathbf{a} = \begin{pmatrix} -4 \\ 7 \\ -4 \end{pmatrix}$.

Find |a| and a unit vector in the direction of a.

$|\mathbf{a}| = \sqrt{4^2 + 7^2 + 4^2} = \sqrt{81} = 9$
Unit vector $= \frac{1}{9}\mathbf{a}$
Answer: |a| = 9, unit vector $\frac{1}{9}\begin{pmatrix} -4 \\ 7 \\ -4 \end{pmatrix}$

10 Reduction to linear form (logarithms) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The variables x and y satisfy $y = ax^n$. A graph of ln y against ln x is a straight line with gradient 1.5 that passes through (0, ln 4).
Find the values of a and n.

$\ln y = \ln a + n \ln x$
gradient = n = 1.5
intercept $\ln a = \ln 4$, so a = 4
Answer: a = 4, n = 1.5