Day 217 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Double angle values from a given ratio (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Given that $\sin\theta = \frac{24}{25}$ and $90° < \theta < 180°$,

find the exact value of $\tan 2\theta$.

$\cos\theta = \pm\sqrt{1 - (\frac{24}{25})^2} = -\frac{7}{25}$ (θ is obtuse, so $\cos\theta < 0$)

$\sin 2\theta = -\frac{336}{625}$ and $\cos 2\theta = -\frac{527}{625}$

$\tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta} = \frac{336}{527}$

Answer: $\frac{336}{527}$

2 Simplifying surds (AQA, Edexcel, OCR, MEI, Eduqas)

Expand and simplify $(-2 - 3\sqrt{7})(-3 + 3\sqrt{7})$, giving your answer in the form $a + b\sqrt{7}$.

$(-2 - 3\sqrt{7})(-3 + 3\sqrt{7}) = 6 - 6\sqrt{7} + 9\sqrt{7} - 9\sqrt{7}\times\sqrt{7}$

$\sqrt{7}\times\sqrt{7} = 7$, so the expression $= -57 + 3\sqrt{7}$

Answer: $-57 + 3\sqrt{7}$

3 Standardising to compare (All boards)

Marks are normal: Maths N(55, 10^2), Physics N(40, 8^2).

Anna scores 75 in Maths and 46 in Physics. Which is her better performance, relatively?

z-score in Maths $= \frac{75 - 55}{10} = 2.00$

z-score in Physics $= \frac{46 - 40}{8} = 0.750$

The higher z-score is in Maths

Answer: Maths

4 Power, driving force and resistance (CIE)

A car of mass 1000 kg moves along a horizontal road. The engine works at a constant rate of 30 kW. The resistance to motion is constant at 1000 N.

Find the acceleration when the speed is 12 m s^{-1}.

$P = Fv$: $F = 30000 \div 12 = 2500$ N

$F - R = ma$: $2500 - 1000 = 1000\,a$

Answer: 1.5 m s^{-2}

5 R cos(θ − α) form: solving (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Given that $3\cos\theta + 4\sin\theta \equiv 5\cos(\theta - 53.1°)$,

solve $3\cos\theta + 4\sin\theta = 2.5$ for $0 \le \theta \le 360°$. (1 d.p.)

$5\cos(\theta - 53.1°) = 2.5$, so $\cos(\theta - 53.1°) = \frac{2.5}{5}$

$\theta - 53.1° = \pm 60.0°$

$\theta = 53.1° + 60.0° = 113.1°$

$\theta = 53.1° - 60.0° = -6.9°$, so $\theta = 353.1°$

Answer: θ = 113.1°, 353.1°

6 Proof by exhaustion (AQA, Edexcel, OCR, MEI, Eduqas)

Prove, by exhaustion, that a square number cannot be written in the form $7k + 3$, where k is an integer.

Write the integer as $n = 7q + j$, where $j = 0, 1, ..., 6$.

n^2 leaves the same remainder as j^2 when divided by 7.

$j = 0, 1, 2, 3, 4, 5, 6$ give remainders 0, 1, 4, 2, 2, 4, 1.

The only possible remainders are 0, 1, 2, 4, never 3.

Answer: possible remainders 0, 1, 2, 4, never 3

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7 Differential equations in context (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A cup of tea cools in a room at 20°C: $\frac{dT}{dt} = -k(T - 20)$. Initially T = 80 and after 8 minutes T = 45.

Find k to 3 s.f. and the time to cool to 30°C, to 3 s.f.

$\frac{1}{T - 20}\, dT = -k\, dt$, so $\ln(T - 20) = -kt + c$, i.e. $T = 20 + 60e^{-kt}$

$t = 8$: $45 = 20 + 60e^{-8k}$, so $e^{-8k} = \frac{5}{12}$

$k = \frac{1}{8} \ln \frac{60}{25} = 0.1094$

$30 = 20 + 60e^{-kt}$: $t = \frac{1}{k} \ln \frac{60}{10} = 16.37$

Answer: k = 0.109, t = 16.4 minutes

8 Binomial: smallest n (All boards)

Given that each trap catches a rabbit independently with probability 0.05, find the least number of traps needed for the probability of at least one success to exceed 0.999.

P(at least one) = $1 - 0.95^n > 0.999$

$0.95^n < 0.001$

$n > \frac{\log 0.001}{\log 0.95} = 134.7$

Answer: n = 135

9 Modulus inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $|2x + 3| > 1$.

$|2x + 3| > 1$ means $2x + 3 < -1$ or $2x + 3 > 1$

Solve each part (dividing by 2): $x < -2$ or $x > -1$

Answer: $x < -2$ or $x > -1$

10 Pulley system: speed on reaching the ground (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Particles A (1 kg) and B (3 kg) are joined by a light string over a smooth pulley. Both are held with the string taut, B is 1.25 m above the ground, then released.

Take g = 9.8 m s^{-2}.

Find the speed with which B hits the ground.

$a = (3 - 1)g \div (3 + 1) = 4.9$

$v^2 = 0 + 2 \times 4.9 \times 1.25 = 12.25$

$v = 3.5$

Answer: 3.5 m s^{-1}