Day 216 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Hypothesis test for correlation (AQA, Edexcel, OCR, MEI, Eduqas)

25 pairs of hours of sleep and reaction time give r = 0.440.
Two-tailed critical value (5%): 0.3961.
Test for correlation.

H_0: $\rho = 0$, H_1: $\rho \neq 0$
$|r| = 0.440 > 0.3961$
r is in the critical region, so reject H_0
Conclusion: there is evidence of correlation between hours of sleep and reaction time
Answer: reject H_0: there is evidence of correlation between hours of sleep and reaction time

2 Rough horizontal surface: finding μ (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A crate of mass 4 kg is pulled along rough horizontal ground by a horizontal force of 13.84 N. It accelerates at 1.5 m s^{-2}.
Take g = 9.8 m s^{-2}.
Find the coefficient of friction between the crate and the ground.

$R = mg = 4 \times 9.8 = 39.2$
Resultant: $13.84 - F = 4 \times 1.5$, so $F = 7.84$
$F = \mu R$: $\mu = 7.84 \div 39.2$
Answer: 0.2

3 Inverse functions (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = \sqrt{2x + 1}$ for $x \geq -\frac{1}{2}$.

Find $f^{-1}(x)$ and state its domain.

Let $y = \sqrt{2x + 1}$. Then $y^2 = 2x + 1$
$x = \frac{y^2 - 1}{2}$
$f(x) \geq 0$, so the domain of f^{-1} is $x \geq 0$.
Answer: $f^{-1}(x) = \frac{x^2 - 1}{2}$, domain $x \geq 0$

4 Resultant force and acceleration (vectors) (AQA, Edexcel, OCR, MEI, Eduqas)

Two forces $(4\mathbf{i} + 8\mathbf{j})$ N and $(2\mathbf{i})$ N act on a particle of mass 5 kg.
Find the magnitude of the acceleration of the particle.

Resultant = $(4\mathbf{i} + 8\mathbf{j}) + (2\mathbf{i}) = 6\mathbf{i} + 8\mathbf{j}$
$|R| = \sqrt{(6^2 + 8^2)} = 10$ N
$a = F/m = 10 \div 5$
Answer: 2 m s^{-2}

5 Integrating composite functions (reverse chain rule) (All boards)

Find (a) $\int e^{2x}\,dx$,

(b) the exact value of $\int_0^1 e^{2x}\,dx$.

(a) Reverse the chain rule and divide by the coefficient of x: $\frac{e^{2x}}{2} + c$

(b) $\left[\frac{e^{2x}}{2}\right]_0^1$

$= \frac{e^2}{2} - \frac{1}{2}$

Answer: (a) $\frac{e^{2x}}{2} + c$ (b) $\frac{e^2}{2} - \frac{1}{2}$

6 Factor theorem with an unknown (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = 2x^3 + kx^2 - 20x - 48$.
Given that $(x + 2)$ is a factor of $f(x)$,
(a) find the value of k,
(b) factorise $f(x)$ completely.

$f(-2) = 0$: $-16 + 4k + 40 - 48 = 0$
$k = 6$
$f(x) = (x + 2)(2x^2 + 2x - 24)$
$= 2(x - 3)(x + 2)(x + 4)$
Answer: (a) $k = 6$ (b) $f(x) = 2(x - 3)(x + 2)(x + 4)$

7 Binomial: smallest n (All boards)

Given that each trap catches a rabbit independently with probability 0.05, find the least number of traps needed for the probability of at least one success to exceed 0.95.

$P(\text{at least one}) = 1 - 0.95^n > 0.95$
$0.95^n < 0.05$
$n > \frac{\log 0.05}{\log 0.95} = 58.40$
Answer: $n = 59$

8 Partial fractions and integration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Write $\frac{-5x - 18}{(x + 3)(x + 4)}$ in partial fractions.

(b) Hence evaluate $\int_0^3 \frac{-5x - 18}{(x + 3)(x + 4)}\,dx$,
giving the answer as a single logarithm.

(a) $\frac{-5x - 18}{(x + 3)(x + 4)} \equiv \frac{A}{x + 3} + \frac{B}{x + 4}$, so $-5x - 18 \equiv A(x + 4) + B(x + 3)$
$x = -3$: $A = -3$; $x = -4$: $B = -2$
(b) $\int_0^3 \left(\frac{-3}{x + 3} + \frac{-2}{x + 4}\right) dx = [-3\ln(x + 3) - 2\ln(x + 4)]_0^3$
$= -3\ln\frac{6}{3} - 2\ln\frac{7}{4} = \ln\left(\frac{2}{49}\right)$

Answer: (a) $A = -3$, $B = -2$ (b) $\ln\left(\frac{2}{49}\right)$

9 Combined inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the set of values of x for which
$4x - 3 \geq -3$ and $x^2 - 9x + 20 \geq 0$.

$4x - 3 \geq -3$ gives $x \geq 0$
$x^2 - 9x + 20 \geq 0$: $(x - 4)(x - 5) \geq 0$ gives $x \leq 4$ or $x \geq 5$
Both must hold: $0 \leq x \leq 4$ or $x \geq 5$
Answer: $0 \leq x \leq 4$ or $x \geq 5$

10 Sector and arc in terms of π (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A sector of a circle has radius 11 cm and area $\frac{363\pi}{4}$ cm^2.

Find the angle of the sector in degrees.

$\frac{1}{2}r^2\theta = \frac{363\pi}{4}$, so $\theta = \frac{2 \times \frac{363\pi}{4}}{11^2} = \frac{3\pi}{2}$ radians
$\theta = \frac{3\pi}{2} \times \frac{180}{\pi} = 270°$
Answer: 270°