**Day 215** **A Level Maths: B Tier 10-a-day: Model Solutions**

## 1 Normal curve and shaded area (All boards)

$X \sim N(70, 5^2)$. The shaded area on the diagram represents the probability that X lies between two values.
Find this probability to 3 significant figures.

Shaded area = P(65.00 < X < 75.00)
= P(X < 75.00) − P(X < 65.00)
= 0.8413 − 0.1587 = 0.683
**Answer: 0.683**

## 2 Speed and direction from a velocity vector (AQA, Edexcel, OCR, MEI, Eduqas)

A boat moves with constant velocity $(-8\mathbf{i} + 15\mathbf{j})$ m s$^{-1}$, where $\mathbf{i}$ and $\mathbf{j}$ are unit vectors east and north.
Find its speed and the acute angle its direction makes with $\mathbf{i}$, to 1 d.p.

Speed = $\sqrt{(-8^2 + 15^2)} = 17$
$\tan\theta = 15 \div 8$
**Answer: 17 m s$^{-1}$; 61.9°**

## 3 Expectation and variance of aX + b (AQA, OCR, MEI, Eduqas, CIE)

The random variable X has E(X) = 8 and Var(X) = 4.
Find (a) E(−3X − 4),
(b) Var(−3X − 4), (c) E($X^2$).

(a) E(aX + b) = aE(X) + b = (−3) × 8 − 4 = −28
(b) Var(aX + b) = $a^2$Var(X) = 9 × 4 = 36
(c) Var(X) = E($X^2$) − [E(X)]$^2$, so E($X^2$) = 4 + 64 = 68
**Answer: (a) −28 (b) 36 (c) 68**

## 4 Integrals of the form f′(x)/f(x) (All boards)

Find the exact value of

$$\int_0^{\frac{\pi}{2}} \frac{\cos x}{\sin x + 3}\,dx,$$

giving your answer in the form a ln b.

The numerator is a multiple of the derivative of the denominator

$$\int \frac{\cos x}{\sin x + 3}\,dx = \ln(\sin x + 3)$$

$$= [\ln(\sin x + 3)]_0^{\frac{\pi}{2}}$$

$$= \ln\left(\frac{4}{3}\right)$$

**Answer: $\ln\left(\frac{4}{3}\right)$**

## 5 Discriminant problems (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Show that the equation $2x^2 + 2x + 3 = 0$ has no real roots.

$b^2 - 4ac = 2^2 - 4 \times 2 \times 3$
$= 4 - 24 = -20$
The discriminant is negative, so there are no real roots.
**Answer: $b^2 - 4ac = -20 < 0$**

## 6 Double angle equations (All boards)

Solve $\cos 2x = 5 \sin x - 2$ for $0 \le x \le 360°$,
giving answers to 1 d.p. where necessary.

Use the double angle formula: $1 - 2\sin^2 x = 5 \sin x - 2$
$(2 \sin x - 1)(\sin x + 3) = 0$
$\sin x = \frac{1}{2}$: x = 30.0°, 150.0°
$\sin x = -3$ has no solutions
**Answer: x = 30.0°, 150.0°**

## 7 Graph of the modulus of a quadratic (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the curve y = f(x), a quadratic.
(a) Find the coordinates of the local maximum point of y = |f(x)|.
(b) How many solutions has |f(x)| = 21?

(a) y = |f(x)| reflects the part of the curve below the x-axis in the x-axis:
(−1, −16) becomes (−1, 16)
(b) The maximum of the middle part is 16. Compare k = 21 with 16:
k > 16 gives 2 intersections with y = k.
**Answer: (a) (−1, 16) (b) 2**

## 8 Particle on a table joined to a hanging particle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle of mass 9 kg lies on a smooth horizontal table. It is attached by a light string over a smooth pulley at the edge of the table to a particle of mass 9 kg hanging freely.
Take g = 9.8 m $s^{-2}$.
The system is released from rest. Find the acceleration and the tension in the string.

Hanging: 9 g − T = 9 a
Table: T = 9 a
9 g = 18 a, so a = 4.9
T = 9 × 4.9
**Answer: a = 4.9 m $s^{-2}$; T = 44.1 N**

## 9 Intersection of two lines in 3D (CIE)

Line $l_1$: $\mathbf{r} = \begin{pmatrix} 0 \\ 4 \\ 10 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ -1 \\ -2 \end{pmatrix}$.

Line $l_2$: $\mathbf{r} = \begin{pmatrix} -4 \\ 5 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ -2 \\ 2 \end{pmatrix}$.

Show that $l_1$ and $l_2$ intersect and find the point.

i: $0 + 0\lambda = -4 + 2\mu$
j: $4 + (-1)\lambda = 5 + (-2)\mu$
Solving i and j: $\lambda = 3$, $\mu = 2$
k check: $10 + (-2)(3) = 4$ and $0 + 2(2) = 4$, equal
Point: $\begin{pmatrix} 0 \\ 1 \\ 4 \end{pmatrix}$
**Answer:** $\begin{pmatrix} \mathbf{0} \\ \mathbf{1} \\ \mathbf{4} \end{pmatrix}$

## 10 Geometric sequences and series (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A geometric series has second term −16 and fifth term 2.
Find (a) the common ratio, (b) the first term, (c) the sum to infinity.

$ar = -16$ and $ar^4 = 2$, so $r^3 = \frac{2}{-16} = -\frac{1}{8}$

$r = -\frac{1}{2}$, and $a = -16 \div (-\frac{1}{2}) = 32$

$S_\infty = \frac{a}{1 - r} = \frac{32}{1 - (-\frac{1}{2})} = \frac{64}{3}$

**Answer: (a) $r = -\frac{1}{2}$ (b) a = 32 (c) $\frac{64}{3}$**