Day 214 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Normal distribution probabilities (All boards)

The mass X g of a flour bag is normal with mean 170, s.d. 8.
Find, to 3 s.f., the probability a value is
(a) more than 162,
(b) between 162 and 170.

$X \sim N(170, 8^2)$
(a) $P(X > 162) = 0.841$
(b) $P(162 < X < 170) = 0.5000 - 0.1587 = 0.341$
Answer: (a) 0.841 (b) 0.341

2 Double angle values from a given ratio (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Given that $\tan\theta = \frac{7}{24}$ and $0 < \theta < 90°$,
find the exact value of $\tan 2\theta$.

Right-angled triangle 7, 24, 25: $\sin\theta = \frac{7}{25}$, $\cos\theta = \frac{24}{25}$

$\sin 2\theta = \frac{336}{625}$ and $\cos 2\theta = \frac{527}{625}$

$\tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta} = \frac{336}{527}$

Answer: $\frac{336}{527}$

3 Equation of a line through two points (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The points A and B have coordinates (−1, 2) and (4, 0).
(a) Find the midpoint of AB.
(b) Find the line AB as ax + by + c = 0, with integer a, b, c.

(a) Midpoint $= \left(\frac{-1+4}{2}, \frac{2+0}{2}\right) = \left(\frac{3}{2}, 1\right)$

(b) Gradient $= \frac{-2}{5} = -\frac{2}{5}$

$y - 2 = -\frac{2}{5}(x + 1)$

Multiply out and collect terms: $2x + 5y - 8 = 0$

Answer: (a) $\left(\frac{3}{2}, 1\right)$ **(b)** $2x + 5y - 8 = 0$

4 Lift: weighing scales reading (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A woman of mass 80 kg stands on weighing scales in a lift. The lift decelerates at 1.5 m s^{-2} while moving upwards.
Take g = 9.8 m s^{-2}.
Find the force exerted on the woman by the scales.

The acceleration is downwards, magnitude 1.5
Taking up as positive: $R - 80 \times 9.8 = -80 \times 1.5$
$R = 80(9.8 - 1.5)$
Answer: 664 N

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5 Collinear points and parallel vectors (All boards)

Points A, B and C have position vectors $\begin{pmatrix}-2\\3\\1\end{pmatrix}$, $\begin{pmatrix}1\\1\\0\end{pmatrix}$ and $\begin{pmatrix}-5\\5\\r\end{pmatrix}$ respectively.

Given that A, B, C are collinear, find r.

$\text{AB} = \begin{pmatrix}3\\-2\\-1\end{pmatrix}$, $\text{AC} = \begin{pmatrix}-3\\2\\r-1\end{pmatrix}$

AC = λAB: component 1 gives $-3 = \lambda \times 3$, so $\lambda = -1$

$r - 1 = -1 \times -1$, so $r = 2$

Answer: r = 2

6 Two independent binomials (All boards)

X ~ B(7, 0.4) and Y ~ B(11, 0.6) are independent.

Find P(X = 3 and Y ≤ 7), to 3 significant figures.

P(X = 3) = 0.2903

P(Y ≤ 7) = 0.7037

Independent, so multiply: 0.2903 × 0.7037 = 0.204

Answer: 0.204

7 Coefficient in a product with a binomial (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the coefficient of x^2 in the expansion of $(2 + 4x)(2 + 3x)^4$.

$(2 + 3x)^4$ has x^1 coefficient 96 and x^2 coefficient 216

x^2 term of the product = 2 × 216 + 4 × 96

= 816

Answer: 816

8 Differential equations in context (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A cup of tea cools in a room at 25°C: $\frac{dT}{dt} = -k(T - 25)$. Initially T = 100 and after 6 minutes T = 55.

Find k to 3 s.f. and the time to cool to 35°C, to 3 s.f.

$\frac{1}{T - 25}\,dT = -k\,dt$, so $\ln(T - 25) = -kt + c$, i.e. $T = 25 + 75e^{-kt}$

$t = 6$: $55 = 25 + 75e^{-6k}$, so $e^{-6k} = \frac{2}{5}$

$k = \frac{1}{6}\ln\frac{75}{30} = 0.1527$

$35 = 25 + 75e^{-kt}$: $t = \frac{1}{k}\ln\frac{75}{10} = 13.19$

Answer: k = 0.153, t = 13.2 minutes

9 Particle supported by two strings (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle of mass 2 kg is suspended in equilibrium by two light strings attached to a horizontal ceiling. The strings make angles α and β with the ceiling, where $\tan\alpha = \frac{3}{4}$ and $\tan\beta = \frac{4}{3}$.

Take $g = 9.8$ m s^{-2}.
Find the tension in each string.

Horizontal: $T_1 \cos\alpha = T_2 \cos\beta$, $T_1 \times \frac{4}{5} = T_2 \times \frac{3}{5}$

Vertical: $T_1 \sin\alpha + T_2 \sin\beta = 19.6$

From the first: $T_1 = \frac{3}{4} T_2$; into the second: $T_2 \times \frac{5}{4} = 19.6$

Answer: $T_1 = 11.76$ N (at α); $T_2 = 15.68$ N (at β)

10 Periodic sequences (AQA, Edexcel, OCR, MEI, Eduqas)

A sequence is defined by $u_{n+1} = \frac{u_n - 1}{u_n + 1}$, $u_1 = -3$.

(a) Show that the sequence is periodic, stating the period.

(b) Find u_{50} and $\sum_{r=1}^{50} u_r$.

(a) $u_2 = 2$, $u_3 = \frac{1}{3}$, $u_4 = -\frac{1}{2}$, $u_5 = -3$

$u_5 = u_1$ so the sequence repeats with period 4

(b) $50 = 4 \times 12 + 2$, so $u_{50} = u_2 = 2$

Sum of one cycle $= -\frac{7}{6}$, so the sum $= 12 \times (-\frac{7}{6}) - 1 = -15$

Answer: period 4; $u_{50} = 2$; sum $= -15$