Day 213 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Projectile at an angle: time and range (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A football is projected from level ground with horizontal velocity component 36 m s^{-1} and vertical velocity component 9.8 m s^{-1} upwards.

Take g = 9.8 m s^{-2}.

Find (a) the time of flight, (b) the horizontal range.

(a) Vertical: $0 = 9.8\,t - 4.9t^2$, so $t = 9.8 \div 4.9$

(b) Range = horizontal speed × time = 36 × 2

Answer: (a) 2 s (b) 72 m

2 Centre and radius of a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A circle has equation $x^2 + y^2 + 2x - 2y - 16 = 0$.

Find the coordinates of its centre and its radius.

Complete the square in x and y:

$(x + 1)^2 - 1 + (y - 1)^2 - 1 - 16 = 0$

$(x + 1)^2 + (y - 1)^2 = 18$

Centre $(-1, 1)$, radius $3\sqrt{2}$

Answer: centre $(-1, 1)$, radius $3\sqrt{2}$

3 Simplify using double angle formulae (All boards)

Simplify $\frac{2 \sin 3x \cos 3x}{\cos 6x}$,

giving your answer as a single trigonometric term.

$2 \sin 3x \cos 3x = \sin 6x$

so the expression is $\frac{\sin 6x}{\cos 6x} = \tan 6x$

Answer: tan 6x

4 Transformations of a turning point (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

y = f(x) has a maximum point at (3, −3).

Find the turning point on the curve:

(a) y = 3f(x) (b) $y = f(\frac{x}{3})$

(c) y = f(x + 2)

(a) y = 3f(x): the point (3, −3) maps to (3, −9)

(b) $y = f(\frac{x}{3})$: the point (3, −3) maps to (9, −3)

(c) y = f(x + 2): the point (3, −3) maps to (1, −3)

Answer: (a) (3, −9) (b) (9, −3) (c) (1, −3)

5 Binomial mean and variance (All boards)

X has a binomial distribution with mean 12 and variance 4.8.

(a) Find n and p.

(b) Find P(X = 12) to 3 significant figures.

(a) $np = 12$ and $np(1 - p) = 4.8$

$1 - p = 4.8 \div 12 = 0.4$, so $p = 0.6$ and $n = 12 \div 0.6 = 20$

(b) $P(X = 12) = {}^{20}C_{12} \times 0.6^{12} \times 0.4^{8} = 0.180$

Answer: (a) n = 20, p = 0.6 (b) 0.180

Day 213 **A Level Maths: B Tier 10-a-day: Model Solutions**

6 Exponential growth and decay models (All boards)

A population is modelled by $N = N_0 e^{kt}$, where t is in hours. Initially N = 500 and after 3 hours N = 1250.
Find (a) k to 3 s.f., (b) when N reaches 4000, to 3 s.f.

$N_0 = 500$, so $1250 = 500\, e^{3k}$, $e^{3k} = 2.5$

(a) $k = \frac{1}{3} \ln 2.5 = 0.3054$

(b) $500\, e^{kt} = 4000$, so $t = \frac{\ln 8}{k} = 6.808$ hours

Answer: (a) k = 0.305 (b) t = 6.81 hours

7 Projectile: greatest height and speed at the top (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A stone is projected from ground level with initial velocity components 39 m s^{-1} horizontally and 49 m s^{-1} vertically upwards.
Take g = 9.8 m s^{-2}.
Find (a) the greatest height, (b) the speed at the greatest height.

(a) $0 = 49^2 - 2 \times 9.8 \times H$, $H = 2401 \div 19.6$
(b) At the top the vertical velocity is zero, so the speed is the horizontal component.
Answer: (a) 122.5 m (b) 39 m s^{-1}

8 Discriminant problems (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The equation $x^2 + 2kx + (k + 6) = 0$ has real roots.
Find the possible values of k.

$b^2 - 4ac = (2k)^2 - 4(k + 6) = 4k^2 - 4k - 24$
$= 4(k^2 - k - 6) = 4(k - 3)(k + 2)$
Need $b^2 - 4ac \geq 0$, so $k \leq -2$ or $k \geq 3$.
Answer: $k \leq -2$ or $k \geq 3$

9 Finding mean and standard deviation from two probabilities (All boards)

The random variable X is normally distributed.
P(X < 48) = 0.0668 and P(X > 68) = 0.1587.
Find the mean and standard deviation (3 s.f.).

$z_1 = -1.500$ and $z_2 = 0.9998$
$48 = \mu + (-1.500)\sigma$ and $68 = \mu + 0.9998\sigma$
Subtract: $20 = 2.500\, \sigma$, so $\sigma = 8.00$
$\mu = 48 - (-1.500)(8.000) = 60.0$
Answer: $\mu = 60.0$, $\sigma = 8.00$

10 Intersection of two lines in 3D (CIE)

Line l_1: $\mathbf{r} = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 3 \\ 0 \end{pmatrix}$.

Line l_2: $\mathbf{r} = \begin{pmatrix} 3 \\ 3 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix}$.

Show that l_1 and l_2 intersect and find the point.

i: $1 + 3\lambda = 3 + 1\mu$

j: $-1 + 3\lambda = 3 + (-1)\mu$

Solving i and j: $\lambda = 1$, $\mu = 1$

k check: $0 + 0(1) = 0$ and $2 + (-2)(1) = 0$, equal

Point: $\begin{pmatrix} 4 \\ 2 \\ 0 \end{pmatrix}$

Answer: $\begin{pmatrix} \mathbf{4} \\ \mathbf{2} \\ \mathbf{0} \end{pmatrix}$