Day 212 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Mean and standard deviation from sums (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A summary of the ages, a years, of 11 members of a club gives $\Sigma a = 323$ and $\Sigma a^2 = 11693$.
Find the mean and the standard deviation (3 s.f.).

Mean = $323 \div 11 = 29.4$

Variance = $\frac{11693}{11} - (29.364)^2 = 200.8$

Standard deviation = $\sqrt{200.8} = 14.2$

Answer: mean 29.4, standard deviation 14.2

2 Speed-time graph: trapezium (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the speed-time graph of a train between two stations.
Find (a) the acceleration in the first 5 s, (b) the distance between the stations.

(a) a = gradient = $16 \div 5$
(b) Distance = area of trapezium = $\frac{1}{2} \times (39 + 30) \times 16$

Answer: (a) 3.2 m s^{-2} (b) 552 m

3 Reciprocal graph: asymptotes and intercepts (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has equation $y = \frac{-3}{x+2} - 1$.

(a) Write down the asymptotes.
(b) Find where the curve crosses the axes.

(a) The denominator is zero when $x = -2$, and $y \to -1$ as $x \to \pm\infty$
Asymptotes: $x = -2$ and $y = -1$

(b) $x = 0$: $y = -\frac{3}{2} - 1 = -\frac{5}{2}$

$y = 0$: $\frac{-3}{x+2} = 1$, so $x + 2 = -3$, $x = -5$

Answer: (a) $x = -2$, $y = -1$ (b) $(0, -\frac{5}{2})$ and $(-5, 0)$

4 Derivatives of tan, sec, cosec and cot (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Differentiate $y = 3 \tan x$ with respect to x,

and find the exact gradient when $x = \frac{\pi}{4}$.

Use the standard results for tan, sec, cosec and cot with the chain rule

$\frac{dy}{dx} = 3 \sec^2 x$

At $x = \frac{\pi}{4}$: $\frac{dy}{dx} = 6$

Answer: $\frac{dy}{dx} = 3 \sec^2 x$; gradient 6

5 Graph of the modulus of a quadratic (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the curve y = f(x), a quadratic.
(a) Find the coordinates of the local maximum point of y = |f(x)|.
(b) How many solutions has |f(x)| = 1?

(a) y = |f(x)| reflects the part of the curve below the x-axis in the x-axis:
(1, −1) becomes (1, 1)
(b) The maximum of the middle part is 1. Compare k = 1 with 1:
k = 1 gives 3 intersections with y = k.
Answer: (a) (1, 1) (b) 3

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6 Work-energy: speed after falling or sliding (CIE)

A bead of mass 0.5 kg is released from rest 22.5 m above the ground and falls freely.
Take g = 9.8 m s^{-2}.
Use energy to find its speed on reaching the ground.

Loss of PE = gain of KE: $m \times 9.8 \times 22.5 = \frac{1}{2} m v^2$
$v^2 = 19.6 \times 22.5 = 441$
$v = 21$
Answer: 21 m s^{-1}

7 Double angle values from a given ratio (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Given that $\tan\theta = -\frac{7}{24}$ and $90° < \theta < 180°$,
find the exact value of $\sin 2\theta$.

Right-angled triangle 7, 24, 25: $\sin\theta = \frac{7}{25}$, $\cos\theta = -\frac{24}{25}$ (θ obtuse)
$\sin 2\theta = 2\sin\theta\cos\theta = 2 \times \frac{7}{25} \times (-\frac{24}{25}) = -\frac{336}{625}$
Answer: $-\frac{336}{625}$

8 Partial fractions (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Express $\frac{x + 28}{(x - 2)(x + 3)}$ in partial fractions.

$\frac{x + 28}{(x - 2)(x + 3)} \equiv \frac{A}{(x - 2)} + \frac{B}{(x + 3)}$, so $x + 28 \equiv A(x + 3) + B(x - 2)$
$x = 2$: $30 = A \times 5$, so $A = 6$
$x = -3$: $25 = B \times (-5)$, so $B = -5$
Answer: $\frac{6}{(x - 2)} - \frac{5}{(x + 3)}$

9 Inverse normal (All boards)

The mass X g of a flour bag is normal with mean 50, s.d. 4.
Find k (3 s.f.): (a) P(X < k) = 0.2,
(b) P(X > k) = 0.01.

(a) $z = -0.8416$, so $k = 50 + (-0.8416) \times 4 = 46.6$
(b) P(X < k) = 0.99, $z = 2.326$, so $k = 50 + 2.326 \times 4 = 59.3$
Answer: (a) 46.6 (b) 59.3

10 Exact values of sec, cosec and cot (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the exact value of cot 60°,
giving your answer with a rational denominator.

$\cot 60° = \frac{1}{\tan 60°}$
$\tan 60° = \sqrt{3}$, so $\cot 60° = \frac{1}{\sqrt{3}}$
Rationalising the denominator gives $\frac{\sqrt{3}}{3}$
Answer: $\frac{\sqrt{3}}{3}$