Day 211 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Standard integrals (definite) (All boards)

Find the exact value of

$\int_0^{\frac{\pi}{6}} (5\sin 3x + \cos 3x)\,dx$.

$\int (5\sin 3x + \cos 3x)\,dx = \frac{\sin 3x}{3} - \frac{5\cos 3x}{3}$

$= [\frac{\sin 3x}{3} - \frac{5\cos 3x}{3}]_0^{\frac{\pi}{6}}$

$= 2$

Answer: 2

2 Frequency table: mean and standard deviation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The table shows the number of siblings, x, of 41 students.
Calculate the mean and standard deviation (3 s.f.).

$\Sigma f = 41$, $\Sigma fx = 83$, $\Sigma fx^2 = 271$

Mean $= \frac{83}{41} = 2.02$

Variance $= \frac{271}{41} - (2.0244)^2 = 2.512$

Standard deviation = 1.58

Answer: mean 2.02, standard deviation 1.58

3 Simplifying algebraic indices (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Simplify $\frac{(2x^2)^2 \times 6x^4}{3x^7}$.

$(2x^2)^2 = 4x^4$

Numerator $= 4 \times 6x^8 = 24x^8$

Divide: $\frac{24}{3} = 8$ and $x^8 \div x^7 = x$

Answer: 8x

4 Integrals of the form f′(x)/f(x) (All boards)

Find the exact value of

$\int_0^{\ln 4} \frac{e^x}{e^x + 2}\,dx$,

giving your answer in the form a ln b.

The numerator is a multiple of the derivative of the denominator

$\int \frac{e^x}{e^x + 2}\,dx = \ln(e^x + 2)$

$= [\ln(e^x + 2)]_0^{\ln 4}$

$= \ln 2$

Answer: ln 2

5 Differentiating displacement (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves on a straight line. Its displacement from O at time t seconds is $s = 2t^3 + t^2 - 8t$ metres.
Find its velocity and acceleration when t = 2.

$v = ds/dt = 6t^2 + 2t - 8$
$a = dv/dt = 12t + 2$
$t = 2$: $v = 20$, $a = 26$

Answer: $v = 20$ m s^{-1}, $a = 26$ m s^{-2}

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6 Two independent binomials (All boards)

X ~ B(12, 0.2) and Y ~ B(12, 0.3) are independent.
Find P(X = 1 and Y ≤ 3), to 3 significant figures.

P(X = 1) = 0.2062
P(Y ≤ 3) = 0.4925
Independent, so multiply: 0.2062 × 0.4925 = 0.102
Answer: 0.102

7 Volume of revolution about the x-axis (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The region R is bounded by the curve $y = x^2 + 3$, the x-axis
and the lines x = 0 and x = 1.
R is rotated through 360° about the x-axis. Find the exact volume generated.

$V = \pi \int_0^1 y^2 \, dx = \pi \int_0^1 (x^4 + 6x^2 + 9) \, dx$

$= \pi[\frac{x^5}{5} + 2x^3 + 9x]_0^1$

$= \frac{56\pi}{5}$

Answer: $\frac{56\pi}{5}$

8 Jumping to a different journey: two-stage motion (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A coach accelerates uniformly from rest at 2 m s^{-2} for 10 s, then travels at constant speed for 20 s, and finally decelerates uniformly at 1 m s^{-2} to rest.
Find the total distance travelled and the total time of the journey.

Max speed = 2 × 10 = 20
Stage 1: ½ × 10 × 20 = 100; stage 2: 20 × 20 = 400
Stage 3: time = 20 ÷ 1 = 20, distance = ½ × 20 × 20 = 200
Answer: 700 m; 50 s

9 Parametric to Cartesian (AQA, Edexcel, OCR, MEI, Eduqas)

A curve has parametric equations
x = 1 + cos t, y = −4 + sin t, for 0 ≤ t < 2π. Find a Cartesian equation of the curve.

cos t = x − 1 and sin t = y + 4
Use $\cos^2 t + \sin^2 t = 1$:
$(x - 1)^2 + (y + 4)^2 = 1$
Answer: $\mathbf{(x - 1)^2 + (y + 4)^2 = 1}$

10 Completing the square (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Write $-x^2 + 2x + 6$ in the form $a(x + p)^2 + q$.
(b) Write down the coordinates of the maximum point of $y = -x^2 + 2x + 6$.

Take out the factor −1: $-x^2 + 2x + 6 = -(x^2 - 2x) + 6$
$= -[(x - 1)^2 - 1] + 6$
$= -(x - 1)^2 + 7$
$(x - 1)^2 \geq 0$ with equality at x = 1, so the maximum is 7 at x = 1.
Answer: (a) $\mathbf{-(x - 1)^2 + 7}$ **(b) (1, 7)**