Day 210 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Resultant of perpendicular forces (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Two forces of magnitude 6 N and 8 N act at right angles on a particle.
Find the magnitude of the resultant, and the angle it makes with the 6 N force, to 1 d.p.

Magnitude = $\sqrt{(6^2 + 8^2)} = \sqrt{100}$
$\tan \alpha = 8 \div 6$
Answer: 10 N at 53.1°

2 Equation of a line through two points (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The points A and B have coordinates $(-6, -5)$ and $(4, 3)$.
(a) Find the midpoint of AB.
(b) Find the line AB as $ax + by + c = 0$, with integer a, b, c.

(a) Midpoint = $(\frac{-6 + 4}{2}, \frac{-5 + 3}{2}) = (-1, -1)$

(b) Gradient = $\frac{8}{10} = \frac{4}{5}$

$y + 5 = \frac{4}{5}(x + 6)$

Multiply out and collect terms: $4x - 5y - 1 = 0$
Answer: (a) $(-1, -1)$ (b) $4x - 5y - 1 = 0$

3 Sector and arc in terms of π (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A sector of a circle has radius 14 cm and angle 120°.
Find the perimeter of the sector, in terms of π.

$\theta = \frac{2\pi}{3}$ radians

Arc length = $r\theta = \frac{28\pi}{3}$ cm

Perimeter = 2r + arc = $28 + \frac{28\pi}{3}$ cm

Answer: $28 + \frac{28\pi}{3}$ cm

4 Binomial expansion (positive integer power) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

In the binomial expansion of $(1 + kx)^n$, where n is a positive integer, the coefficient of x is 12 and the coefficient of x^2 is 54.
Find the values of n and k.

$nk = 12$, so $k = \frac{12}{n}$

$\frac{n(n-1)}{2}k^2 = 54$, so $\frac{n(n-1)}{2} \times \frac{144}{n^2} = 54$

$\frac{(n-1) \times 144}{2n} = 54$, so $144(n - 1) = 108n$

$n = 4$, and $k = 3$
Answer: $n = 4$, $k = 3$

5 Stationary points of exponential and log curves (All boards)

Find the coordinates of the stationary point of the curve $y = 2x - \ln x$,
and determine its nature.

$\frac{dy}{dx} = \frac{2x - 1}{x} = 0$

$x = \frac{1}{2}$

$\frac{d^2y}{dx^2} = \frac{1}{x^2}$

At $x = \frac{1}{2}$: $\frac{d^2y}{dx^2} = 4 > 0$, so a minimum

Answer: $(\frac{1}{2}, \ln 2 + 1)$ minimum

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6 Constant acceleration, two stages of working (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A cyclist moving in a straight line with constant acceleration increases its speed from 15 m s^{-1} to 21 m s^{-1} while covering 54 m.
Find (a) the acceleration, (b) the time taken.

(a) $v^2 = u^2 + 2as$: $441 = 225 + 108\,a$, so $a = 2$
(b) $v = u + at$: $21 = 15 + 2\,t$
$t = 6 \div 2 = 3$
Answer: (a) 2 m s^{-2} (b) 3 s

7 Binomial critical region (All boards)

X ~ B(50, p). H_0: p = 0.25 is to be tested against H_1: p > 0.25 at the 10% significance level.
Find the critical region and the actual significance level.

Under H_0, X ~ B(50, 0.25)
$P(X \geq 17) = 0.0983 \leq 0.1$
$P(X \geq 16) = 0.1631 > 0.1$
The critical region is $X \geq 17$
Answer: $X \geq 17$; actual significance level 9.83%

8 Quadratics in a^x (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve the equation $3^{2x} - 4\times3^x + 3 = 0$.

Let $y = 3^x$. Then $3^{2x} = y^2$, so $y^2 - 4y + 3 = 0$
$(y - 3)(y - 1) = 0$
$y = 3$ or $y = 1$
$3^x = 3$ gives $x = 1$; $3^x = 1$ gives $x = 0$.
Answer: x = 0 or x = 1

9 R cos(θ − α) form: express and maximum (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Given that $12\cos\theta + 5\sin\theta \equiv 13\cos(\theta - 22.6°)$,
find the greatest value of $\frac{1}{(16 + 12\cos\theta + 5\sin\theta)}$.

Denominator is smallest when $13\cos(\theta - 22.6°) = -13$
Smallest denominator = 16 − 13 = 3
Greatest value = $\frac{1}{3}$
Answer: $\frac{1}{3}$

10 Normal distribution probabilities (All boards)

A battery lasts X hours, normal with mean 50, s.d. 1.5.
Find, to 3 s.f., the probability a value is
(a) more than 51.5,
(b) between 51.5 and 52.2.

$X \sim N(50, 1.5^2)$
(a) $P(X > 51.5) = 0.159$
(b) $P(51.5 < X < 52.2) = 0.9288 - 0.8413 = 0.0874$
Answer: (a) 0.159 (b) 0.0874