Day 21 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Simplifying algebraic indices (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Simplify $\frac{(4x)^3 \times 2x}{4x^6}$.

$(4x)^3 = 64x^3$

Numerator $= 64 \times 2x^4 = 128x^4$

Divide: $\frac{128}{4} = 32$ and $x^4 \div x^6 = x^{-2} = \frac{1}{x^2}$

Answer: $\mathbf{\frac{32}{x^2}}$

2 Systematic sampling (AQA, Edexcel, OCR, MEI, Eduqas)

Items numbered 1 to 96; systematic sample of size 16.

(a) Find the sampling interval.

(b) The first item is number 3. Find the 7-th item.

(a) Interval $= 96 \div 16 = 6$

(b) 7-th item $= 3 + 6 \times 6 = 39$

Answer: interval 6; item number 39

3 Trapezium rule (AQA, Edexcel, OCR, MEI, Eduqas)

The diagram shows the trapezium rule with 4 strips.

Use it to estimate $\int_0^2 \sqrt{1 + x^3}\,dx$, to 3 s.f.

$h = \frac{2 - 0}{4} = 0.5$

x: 0, 0.5, 1, 1.5, 2

y: 1.000, 1.061, 1.414, 2.092, 3.000

Area $\approx \frac{h}{2}[y_0 + y_4 + 2(y_1 + \ldots + y_3)]$

$= \frac{0.5}{2} \times [1.000 + 3.000 + 2 \times 4.567] = 3.283$

Answer: 3.28

4 Algebraic division (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the quotient and the remainder when

$2x^3 + 12x^2 - 2x - 10$ is divided by $(2x + 2)$.

$x^2 \times (2x + 2) = 2x^3 + 2x^2$, leaving $10x^2 - 2x - 10$

$5x \times (2x + 2) = 10x^2 + 10x$, leaving $-12x - 10$

$-6 \times (2x + 2) = -12x - 12$, leaving 2

Quotient $x^2 + 5x - 6$, remainder 2

Answer: quotient $\mathbf{x^2 + 5x - 6}$, remainder 2

5 Horizontal projection from a height (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A ball is thrown horizontally at 28 m s^{-1} from the top of a cliff 78.4 m above level ground.

Take $g = 9.8$ m s^{-2}.

Find the time of flight and the horizontal distance from the base where it lands.

Vertical: $78.4 = \frac{1}{2} \times 9.8 \times t^2$, $t^2 = 16$

$t = 4$ s

Horizontal: $x = 28 \times 4$

Answer: 4 s, 112 m

6 Venn diagram with independent events (All boards)

The Venn diagram shows probabilities for events A and B, which are independent.
Find the values of x and y.

P(A) = 0.1875 + 0.0625 = 0.25 and P(A ∩ B) = 0.0625
Independent: P(A ∩ B) = P(A)P(B), so P(B) = 0.0625 ÷ 0.25 = 0.25
x = P(B) − P(A ∩ B) = 0.25 − 0.0625 = 0.1875
y = 1 − (0.1875 + 0.0625 + 0.1875) = 0.5625
Answer: x = 0.1875, y = 0.5625

7 Projectile at an angle: time and range (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A stone is projected from level ground with horizontal velocity component 12 m s^{-1} and vertical velocity component 9.8 m s^{-1} upwards.
Take g = 9.8 m s^{-2}.
Find (a) the time of flight, (b) the horizontal range.

(a) Vertical: $0 = 9.8t - 4.9t^2$, so $t = 9.8 \div 4.9$
(b) Range = horizontal speed × time = 12 × 2
Answer: (a) 2 s (b) 24 m

8 Quadratics in a^x (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve the equation $2^{2x} + 2\times2^x - 8 = 0$.

Let $y = 2^x$. Then $2^{2x} = y^2$, so $y^2 + 2y - 8 = 0$
$(y - 2)(y + 4) = 0$
$y = 2$ or $y = -4$
$2^x > 0$ for all x, so $y = -4$ is rejected. $2^x = 2$ gives $x = 1$.
Answer: x = 1

9 Parametric differentiation: tangents (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has parametric equations
$x = t^3$, $y = t^2 + 2t$.
Find $\frac{dy}{dx}$ in terms of t, and the equation of the tangent at t = 2.

$\frac{dx}{dt} = 3t^2$, $\frac{dy}{dt} = 2t + 2$

$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2(t+1)}{3t^2}$; at t = 2 it is $\frac{1}{2}$

Point: (8, 8)

$y - 8 = \frac{1}{2}(x - 8)$, so $y = \frac{1}{2}x + 4$

Answer: $\frac{dy}{dx} = \frac{2(t+1)}{3t^2}$; tangent $y = \frac{1}{2}x + 4$

10 Rates of change (connected rates) (All boards)

The radius of a circular oil slick increases at 0.25 cm per second.
Find the rate at which its area is increasing when r = 4 cm, in terms of π.

$A = \pi r^2$, so $\frac{dA}{dr} = 2\pi r$

$\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt} = 2\pi r \times 0.25$

When r = 4: $\frac{dA}{dt} = 2\pi$

Answer: 2π cm^2/s