**Day 209** **A Level Maths: B Tier 10-a-day: Model Solutions**

## 1 Integrating velocity (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves in a straight line. At time t seconds its velocity is $v = 3t^2 + 8t + 6$ m s$^{-1}$.
Find the distance moved between t = 1 and t = 4. (The particle does not change direction.)

$s = \int_1^4 (3t^2 + 8t + 6)\,dt$

$= [\,t^3 + 4t^2 + 6t\,]$ from t = 1 to t = 4

$= 152 - 11$

**Answer: 141 m**

## 2 Quadratic inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $x^2 - 4 \leq 0$.

Factorise: $(x - 2)(x + 2) \leq 0$

Critical values: $x = -2$ and $x = 2$

Sketch the ∪-shaped parabola: $-2 \leq x \leq 2$

**Answer: $-2 \leq x \leq 2$**

## 3 Smooth inclined plane (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A block slides from rest down a line of greatest slope of a smooth plane inclined at α, where $\sin\alpha = \frac{3}{5}$ to the horizontal.

Take g = 9.8 m s$^{-2}$.
Find its acceleration, and the distance it travels in 4 s.

Along the plane: $mg\sin\alpha = ma$, so $a = 9.8 \times 0.6 = 5.88$

$s = \tfrac{1}{2}at^2 = \tfrac{1}{2} \times 5.88 \times 4^2$

**Answer: 5.88 m s$^{-2}$; 47.04 m**

## 4 Rationalising the denominator (AQA, Edexcel, OCR, MEI, Eduqas)

Express $\frac{2 + 4\sqrt{7}}{4 + 2\sqrt{7}}$ in the form $p + q\sqrt{7}$, where p and q are integers.

Multiply top and bottom by the conjugate $(4 - 2\sqrt{7})$:

Denominator: $(4 + 2\sqrt{7})(4 - 2\sqrt{7}) = 4^2 - 2^2 \times 7 = -12$

Numerator: $(2 + 4\sqrt{7})(4 - 2\sqrt{7}) = -48 + 12\sqrt{7}$

So the fraction $= \frac{-48 + 12\sqrt{7}}{-12} = 4 - \sqrt{7}$

**Answer: $4 - \sqrt{7}$**

## 5 Area under a curve (All boards)

The diagram shows the curve $y = x(7 - x)$ and the line $y = 2x$.
Find the exact area of the shaded region R.

Intersect: $x(7 - x) = 2x$, so $x = 0$ or $x = 5$

Area $= \int_0^5 (x(7 - x) - 2x)\,dx = \int_0^5 (5x - x^2)\,dx$

$= [\frac{5x^2}{2} - \frac{x^3}{3}]_0^5 = \frac{125}{6}$

**Answer: $\frac{125}{6}$**

## 6 Hypothesis test for correlation (AQA, Edexcel, OCR, MEI, Eduqas)

8 pairs of height and arm span give r = 0.839.
One-tailed critical value (1%): 0.7887.
Test for positive correlation.

$H_0$: $\rho = 0$, $H_1$: $\rho > 0$
r = 0.839, critical value 0.7887; r is beyond the critical value in the right direction
r is in the critical region, so reject $H_0$
Conclusion: there is evidence of positive correlation between height and arm span
**Answer: reject $H_0$: there is evidence of positive correlation between height and arm span**

## 7 Laws of logarithms: solving equations (All boards)

Solve $\log_3 x + \log_3(x + 6) = 3$.

$\log_3(x(x + 6)) = 3$, so $x(x + 6) = 3^3 = 27$
$x^2 + 6x - 27 = 0$, so $(x - 3)(x + 9) = 0$
$x > 0$ for the logs to exist, so $x = 3$
**Answer: $x = 3$**

## 8 Normal approximation to the binomial (AQA, OCR, CIE)

X ~ B(150, 0.3). Use a normal approximation to find $P(41 \leq X \leq 45)$, to 3 significant figures.

$Y \sim N(np, np(1 - p)) = N(45.00, 31.50)$, so $\sigma = \sqrt{31.50} = 5.612$
Use a continuity correction.
$P(41 \leq X \leq 45) \approx P(40.5 < Y < 45.5) = 0.324$
**Answer: 0.324**

## 9 Parametric differentiation: tangents (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has parametric equations $x = 2\cos t$, $y = 5\sin t$.
Find $\frac{dy}{dx}$ in terms of t, and its exact value when $t = \frac{\pi}{6}$.

$\frac{dx}{dt} = -2\sin t$, $\frac{dy}{dt} = 5\cos t$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = -\frac{5\cot t}{2}$$

At $t = \frac{\pi}{6}$: $\frac{dy}{dx} = -\frac{5\sqrt{3}}{2}$

**Answer: $\frac{dy}{dx} = -\frac{5\cot t}{2}$; value $-\frac{5\sqrt{3}}{2}$**

## 10 Chord of a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line $y = -2x - 27$ meets the circle $(x + 2)^2 + (y + 3)^2 = 100$ at the points A and B. Find the coordinates of A and B, and the exact length of the chord AB.

Substitute $y = -2x - 27$: $(x + 2)^2 + (-2x - 24)^2 = 100$
$x^2 + 20x + 96 = 0$
$(x + 8)(x + 12) = 0$, so $x = -12$ or $x = -8$
$y = -2x - 27$: A = (−12, −3), B = (−8, −11)
$AB^2 = 4^2 + (-8)^2 = 80$
**Answer: A(−12, −3), B(−8, −11), $AB = 4\sqrt{5}$**