Day 208 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Mean and standard deviation from sums (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A summary of the times, t minutes, taken by 8 people to complete a puzzle gives $\Sigma t = 194$ and $\Sigma t^2 = 6050$. Find the mean and the standard deviation (3 s.f.).

Mean = $194 \div 8 = 24.2$

Variance = $\frac{6050}{8} - (24.250)^2 = 168.2$

Standard deviation = $\sqrt{168.2} = 13.0$

Answer: mean 24.2, standard deviation 13.0

2 Inequalities with exponents and logs (All boards)

Solve $7^x < 231$, giving your answer to 3 s.f.

Take logs: $x \ln 7 < \ln 231$

$x < \frac{\ln 231}{\ln 7} = 2.797$

Answer: $x < 2.80$

3 Braking distance and time (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A lorry travelling at 24 m s^{-1} brakes and comes to rest with constant deceleration 1 m s^{-2}. Find (a) the distance travelled while braking, (b) the time taken to stop.

(a) $0 = 24^2 - 2 \times 1 \times s$

$s = 576 \div 2 = 288$

(b) $0 = 24 - 1\,t$, so $t = 24 \div 1$

Answer: (a) 288 m (b) 24 s

4 Implicit differentiation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has equation $3x^2y + xy + 3y^3 = 24$.

Find $\frac{dy}{dx}$ in terms of x and y, and the gradient at (0, 2).

Differentiate with respect to x: $3x^2\frac{dy}{dx} + 6xy + x\frac{dy}{dx} + 9y^2\frac{dy}{dx} + y = 0$

$\frac{dy}{dx} = \frac{y(-6x - 1)}{3x^2 + x + 9y^2}$

At (0, 2): gradient = $-\frac{1}{18}$

Answer: $\frac{dy}{dx} = \frac{y(-6x - 1)}{3x^2 + x + 9y^2}$; gradient $-\frac{1}{18}$

5 Binomial one-tailed test (All boards)

A player says she scores with probability 0.3. In 30 shots she scores 7 times. Test, at the 5% level, whether the true proportion is lower than claimed.

H_0: $p = 0.3$, H_1: $p < 0.3$. Let $X \sim B(30, 0.3)$ under H_0

$P(X \leq 7) = 0.2814$

$0.2814 > 0.05$, so do not reject H_0

There is insufficient evidence that the proportion is lower than claimed

Answer: do not reject H_0: insufficient evidence that the proportion is lower

6 Standard integrals (definite) (All boards)

Find the exact value of

$\int_1^3 (x^2 + \frac{4}{x})\,dx$.

$\int (x^2 + \frac{4}{x})\,dx = \frac{x^3}{3} + 4\ln x$

$= [\frac{x^3}{3} + 4\ln x]_1^3$

$= \ln 81 + \frac{26}{3}$

Answer: $\ln 81 + \frac{26}{3}$

7 Chord of a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line $y = -8x + 3$ meets the circle $x^2 + (y - 3)^2 = 65$ at the points A and B. Find the coordinates of A and B, and the exact length of the chord AB.

Substitute $y = -8x + 3$: $x^2 + (-8x)^2 = 65$

$x^2 - 1 = 0$

$(x - 1)(x + 1) = 0$, so $x = -1$ or $x = 1$

$y = -8x + 3$: A = $(-1, 11)$, B = $(1, -5)$

$AB^2 = 2^2 + (-16)^2 = 260$

Answer: A$(-1, 11)$, B$(1, -5)$, AB = $2\sqrt{65}$

8 Inclined plane: force to hold or move the particle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A block of mass 10 kg is held on a rough plane ($\tan\alpha = \frac{3}{4}$, $\mu = 0.5$) by a force P N up the plane, as shown.

Take $g = 9.8$ m s^{-2}. The block is about to move down the plane. Find P.

$R = mg\cos\alpha = 98 \times 4/5 = 78.4$

Friction (limiting) $F = \mu R = 39.2$

Moving down, friction acts up: $P + F = mg\sin\alpha$

$mg\sin\alpha = 58.8$

Answer: 19.6 N

9 Modulus inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $|4x + 6| < 5$.

$|4x + 6| < 5$ means $-5 < 4x + 6 < 5$

Subtract 6 and divide by 4: $-\frac{11}{4} < x < -\frac{1}{4}$

Answer: $-\frac{11}{4} < x < -\frac{1}{4}$

10 Circle through two points (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A circle passes through A$(-7, -2)$ and B$(-4, 7)$, and its centre lies on the x-axis. Find the centre and the equation of the circle.

Let the centre be $(h, 0)$. Then $CA^2 = CB^2$:

$(h + 7)^2 + 4 = (h + 4)^2 + 49$

Expanding and cancelling h^2: $14h + 53 = 8h + 65$

$h = 2$

$r^2 = CA^2 = 85$

Answer: centre $(2, 0)$, $(x - 2)^2 + y^2 = 85$