Day 207 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Speed and direction from a velocity vector (AQA, Edexcel, OCR, MEI, Eduqas)

A remote-control car moves with constant velocity $(-10\mathbf{i} - 24\mathbf{j})$ m s^{-1}, where $\mathbf{i}$ and $\mathbf{j}$ are unit vectors east and north. Find its speed and the acute angle its direction makes with $\mathbf{i}$, to 1 d.p.

Speed = $\sqrt{(-10^2 + -24^2)} = 26$
$\tan\theta = 24 \div 10$
Answer: 26 m s^{-1}; 67.4°

2 Proof using parity (AQA, Edexcel, OCR, MEI, Eduqas)

Prove that if n is odd then $n^2 + n + 7$ is odd.

Let $n = 2k + 1$, where k is an integer.
$n^2 = 4k^2 + 4k + 1$
$n = 2k + 1$
Total = $4k^2 + 6k + 9$
$4k + 6$ is even and 9 is odd, so the total is odd.
Answer: the expression is odd

3 Equilibrium of three forces (components) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle is in equilibrium under three forces: P N at angle θ above the positive x-axis, a force of 6 N in the negative x-direction and a force of Q N in the negative y-direction.

$\cos\theta = \frac{3}{5}$. Find P and Q.

Resolve horizontally: $P\cos\theta = 6$, so $P \times \frac{3}{5} = 6$

$P = 10$

Resolve vertically: $Q = P\sin\theta = 10 \times \frac{4}{5}$

Answer: P = 10 N; Q = 8 N

4 Collinear points and parallel vectors (All boards)

Points A, B and C have position vectors $\begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix}$, $\begin{pmatrix} 0 \\ -1 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 5 \\ r \end{pmatrix}$ respectively.

Given that A, B, C are collinear, find r.

$\text{AB} = \begin{pmatrix} 0 \\ -3 \\ 3 \end{pmatrix}$, $\text{AC} = \begin{pmatrix} 0 \\ 3 \\ r - -1 \end{pmatrix}$

$\text{AC} = \lambda\text{AB}$: component 2 gives $3 = \lambda \times -3$, so $\lambda = -1$
$r - -1 = -1 \times 3$, so $r = -4$
Answer: r = −4

5 Poisson: changing the interval (CIE)

Faults occur at random in a cable at an average rate of 3 per hour of operation. Find the probability that in 20 minutes there are at least 2 (3 s.f.).

Rate in 20 minutes: $\lambda = 3 \times \frac{20}{60} = 1$

$X \sim \text{Po}(1)$
$P(X \geq 2) = 1 - P(X \leq 1) = 0.264$
Answer: 0.264

6 Separable differential equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find y in terms of x, given that $\frac{dy}{dx} = 4y^2e^x$

and y = 0.25 when x = 0.

$\frac{1}{y^2}\,dy = 4e^x\,dx$, so $-\frac{1}{y} = 4e^x + c$

x = 0, y = $\frac{1}{4}$: $-4 = 4 + c$, so $c = -8$

$\frac{1}{y} = 8 - 4e^x$, so $y = \frac{1}{8 - 4e^x}$

Answer: $y = \frac{1}{(8 - 4e^x)}$

7 Binomial: smallest n (All boards)

Given that each trap catches a rabbit independently with probability 0.25, find the least number of traps needed for the probability of at least one success to exceed 0.999.

P(at least one) = $1 - 0.75^n > 0.999$

$0.75^n < 0.001$

$n > \frac{\log 0.001}{\log 0.75} = 24.01$

Answer: n = 25

8 Combined inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the set of values of x for which

$4x - 2 > 2$ and $x^2 - 5x + 4 \leq 0$.

$4x - 2 > 2$ gives $x > 1$

$x^2 - 5x + 4 \leq 0$: $(x - 1)(x - 4) \leq 0$ gives $1 \leq x \leq 4$

Both must hold: $1 < x \leq 4$

Answer: $1 < x \leq 4$

9 Intersection of two lines in 3D (CIE)

Line l_1: $\mathbf{r} = \begin{pmatrix}3\\1\\6\end{pmatrix} + \lambda\begin{pmatrix}1\\3\\2\end{pmatrix}$.

Line l_2: $\mathbf{r} = \begin{pmatrix}-2\\-2\\6\end{pmatrix} + \mu\begin{pmatrix}2\\0\\-1\end{pmatrix}$.

Show that l_1 and l_2 intersect and find the point.

i: $3 + 1\lambda = -2 + 2\mu$

j: $1 + 3\lambda = -2 + 0\mu$

Solving i and j: $\lambda = -1$, $\mu = 2$

k check: $6 + 2(-1) = 4$ and $6 + (-1)(2) = 4$, equal

Point: $\begin{pmatrix}2\\-2\\4\end{pmatrix}$

Answer: $\begin{pmatrix}\mathbf{2}\\\mathbf{-2}\\\mathbf{4}\end{pmatrix}$

10 Proof by contradiction (AQA, Edexcel, OCR, MEI, Eduqas)

Prove by contradiction that there are no integers m and n such that 15m + 10n = 62.

Assume there are integers m and n with 15m + 10n = 62.
15m + 10n = 5(3m + 2n).
So the left-hand side is a multiple of 5.
But 62 = 5 × 12 + 2 is not a multiple of 5.
This is a contradiction, so no such integers exist.
Answer: no integer solutions (LHS is a multiple of 5, 62 is not)