Day 206 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Proof by deduction (AQA, Edexcel, OCR, MEI, Eduqas)

Prove that $(4n + 4)^2 - (4n - 4)^2$ is a multiple of 64 for all integer values of n.

$(4n + 4)^2 = 16n^2 + 32n + 16$

$(4n - 4)^2 = 16n^2 - 32n + 16$

Difference = $64n = 64 \times n$

n is an integer, so this is a multiple of 64 for every integer n.

Answer: difference = 64n, a multiple of 64

2 Basic trig equations (degrees) (All boards)

Solve $\sin x = -\frac{1}{2}$ for $0 \leq x \leq 360°$.

Principal value $x = -30°$

Second solution: $180° - -30° = 210°$

Answer: x = 210°, 330°

3 Impulse of a constant force: v = u + at with forces (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A trolley of mass 4 kg is moving on a smooth horizontal surface at 0 m s^{-1} when a constant horizontal force of 15 N, in the direction of motion, acts for 4 s.

Find the speed of the trolley at the end of this time and the distance it moves.

F = ma: $a = 15 \div 4 = 3.75$

$v = u + at = 0 + 3.75 \times 4$

$s = ut + \frac{1}{2}at^2$

Answer: 15 m s^{-1}; 30 m

4 Compound angle formulae (exact values) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Given that $\sin A = \frac{24}{25}$ and $\cos B = \frac{24}{25}$, where A is obtuse and B is acute,

find the exact value of $\cos(A - B)$.

$\cos A = -\frac{7}{25}$ and $\sin B = \frac{7}{25}$

$\cos(A - B) = \cos A \cos B + \sin A \sin B$

$= (-\frac{7}{25}) \times \frac{24}{25} + \frac{24}{25} \times \frac{7}{25} = 0$

Answer: 0

5 Product moment correlation coefficient (AQA, Edexcel, OCR, MEI, Eduqas)

For 14 pairs of data, $S_{xx} = 90$, $S_{yy} = 320$ and $S_{xy} = 91$.

Find the PMCC (3 s.f.) and describe the correlation.

$r = \frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}} = \frac{91}{\sqrt{90 \times 320}}$

$= \frac{91}{169.71} = 0.536$

r is positive, so there is weak positive correlation

Answer: r = 0.536, weak positive correlation

6 Locating roots: change of sign (All boards)

The function $f(x) = x^3 - 2x - 10$.

Show that f(x) = 0 has a root between x = 2 and x = 2.5.

$f(2) = -6.00$

$f(2.5) = 0.625$

f is continuous and changes sign, so there is a root in the interval

Answer: f(2) < 0 and f(2.5) > 0: sign change

7 Rationalising the denominator (AQA, Edexcel, OCR, MEI, Eduqas)

Express $\frac{15 - 5\sqrt{5}}{\sqrt{5}}$ in the form $p + q\sqrt{5}$, where p and q are integers.

Multiply top and bottom by $\sqrt{5}$:

$\frac{(15 - 5\sqrt{5})\sqrt{5}}{5} = \frac{15\sqrt{5} - 25}{5}$

$= -5 + 3\sqrt{5}$

Answer: $-5 + 3\sqrt{5}$

8 Binomial two-tailed test (All boards)

A coin is thought to land heads with probability 0.2. It is tossed 30 times and lands heads 0 times.
Test, at the 5% significance level, whether the probability of heads is different from 0.2.

H_0: $p = 0.2$, H_1: $p \neq 0.2$. Under H_0, $X \sim B(30, 0.2)$

$P(X \leq 0) = 0.0012$

Two-tailed: compare with 0.025. $0.0012 < 0.025$, so reject H_0

There is sufficient evidence that the probability of heads is different from 0.2

Answer: reject H_0: sufficient evidence that the probability of heads is different from 0.2

9 Combined inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the set of values of x for which

$5x - 3 \geq -1$ and $x^2 + x - 6 \geq 0$.

$5x - 3 \geq -1$ gives $x \geq \frac{2}{5}$

$x^2 + x - 6 \geq 0$: $(x + 3)(x - 2) \geq 0$ gives $x \leq -3$ or $x \geq 2$

Both must hold: $x \geq 2$

Answer: $x \geq 2$

10 Rod suspended by two vertical strings (AQA, Edexcel, OCR, MEI, Eduqas)

A uniform rod AB of length 8 m and mass 10 kg hangs horizontally from two vertical strings attached 0.5 m from A and 0.5 m from B.

A particle of mass 3 kg is attached 2 m from A. Take $g = 9.8$ m s^{-2}. Find the tension in each string.

Moments about the string near A: $T_2(7) = 10\,g(3.5) + 3\,g(1.5)$

$T_2 = 55.3$

$T_1 + T_2 = 13\,g$ so $T_1 = 127.4 - 55.3$

Answer: Near A: 72.1 N; near B: 55.3 N