Day 205 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Exponential and power models (log-linear) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Data are modelled by $y = ax^n$. A plot of $\log_{10} y$ against $\log_{10} x$ is a straight line with intercept 0.778 and gradient 2.
Find a (2 s.f.) and n, then estimate y when x = 4 (3 s.f.).

$\log_{10} y = \log_{10} a + n \log_{10} x$

n = gradient = 2; $\log_{10} a = 0.778$, so $a = 10^{0.778} = 6.0$

$y = 5.998 \times 4^2 = 96.0$

Answer: a = 6.0, n = 2, y = 96.0

2 Describing transformations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Describe the single transformation that maps the graph of y = f(x) onto the graph of:
(a) y = f(2x)
(b) y = −f(x)
(c) y = f(x + 5)

(a) y = f(2x) is a stretch, scale factor $\frac{1}{2}$, parallel to the x-axis.

(b) y = −f(x) is a reflection in the x-axis.

(c) y = f(x + 5) is a translation $\begin{pmatrix} -5 \\ 0 \end{pmatrix}$.

Answer: (a) stretch, scale factor $\frac{1}{2}$, parallel to the x-axis (b) reflection in the x-axis (c) translation $\begin{pmatrix} -5 \\ 0 \end{pmatrix}$

3 Exact values of sec, cosec and cot (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the exact value of cosec 135°,
giving your answer with a rational denominator.

$\text{cosec } 135° = \frac{1}{\sin 135°}$

$\sin 135° = \frac{\sqrt{2}}{2}$, so $\text{cosec } 135° = \frac{1}{\frac{\sqrt{2}}{2}}$

Rationalising the denominator gives $\sqrt{2}$

Answer: $\sqrt{2}$

4 Braking distance and time (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A lorry travelling at 8 m s^{-1} brakes and comes to rest with constant deceleration 1 m s^{-2}.
Find (a) the distance travelled while braking, (b) the time taken to stop.

(a) $0 = 8^2 - 2 \times 1 \times s$
$s = 64 \div 2 = 32$
(b) $0 = 8 - 1\,t$, so $t = 8 \div 1$
Answer: (a) 32 m (b) 8 s

5 Completing the square (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Write $-x^2 - 12x - 37$ in the form $a(x + p)^2 + q$.
(b) Write down the coordinates of the maximum point of $y = -x^2 - 12x - 37$.

Take out the factor −1: $-x^2 - 12x - 37 = -(x^2 + 12x) - 37$
$= -[(x + 6)^2 - 36] - 37$
$= -(x + 6)^2 - 1$
$(x + 6)^2 \geq 0$ with equality at x = −6, so the maximum is −1 at x = −6.
Answer: (a) $-(x + 6)^2 - 1$ (b) (−6, −1)

6 Integration by substitution (given substitution) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Use the substitution $u = e^x + 2$ to find the exact value of

$\int_0^{\ln 2} (e^x + 2)^3 e^x \, dx$.

$\frac{du}{dx} = e^x$, so $e^x \, dx = du$

Limits: $x = 0$ gives $u = 3$, $x = \ln 2$ gives $u = 4$

$\int_3^4 u^3 \, du = [\frac{u^4}{4}]_3^4$

$= \frac{175}{4}$

Answer: $\frac{175}{4}$

7 Hypothesis test for a normal mean (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Packet masses are normal, s.d. 6 g. Claimed mean 120 g; 25 packets have mean 122.5 g.
Test at the 5% level whether the true mean is greater than 120.

$H_0: \mu = 120$, $H_1: \mu > 120$. Under H_0, $\overline{X} \sim N(120, \frac{6^2}{25})$

$z = \frac{122.5 - 120}{6/\sqrt{25}} = 2.083$

$P(Z > 2.083) = 0.0186 < 0.05$

Reject H_0: there is sufficient evidence that the mean has increased

Answer: reject H_0: sufficient evidence of an increase in the mean

8 Particle on a table joined to a hanging particle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle of mass 1 kg lies on a smooth horizontal table. It is attached by a light string over a smooth pulley at the edge of the table to a particle of mass 1 kg hanging freely.

Take $g = 9.8$ m s^{-2}.

The system is released from rest. Find the acceleration and the tension in the string.

Hanging: $1g - T = 1a$

Table: $T = 1a$

$1g = 2a$, so $a = 4.9$

$T = 1 \times 4.9$

Answer: $a = 4.9$ m s^{-2}; $T = 4.9$ N

9 Equations with sec, cosec and cot (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $\tan^2 x + \sec x - 1 = 0$ for $0 \leq x \leq 360°$,
giving answers to 1 d.p.

Use $\tan^2 x = \sec^2 x - 1$: $\sec^2 x + \sec x - 2 = 0$

$(\sec x - 1)(\sec x + 2) = 0$, so $\sec x = 1$ or -2

$\sec x = 1$, so $\cos x = 1$: $x =$

$\sec x = -2$, so $\cos x = -\frac{1}{2}$: $x = 120.0°, 240.0°$

Answer: $x = 120.0°, 240.0°$

10 Geometric series and logarithms (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A geometric sequence has first term 50 and common ratio 0.75.
Find the smallest n for which the nth term is less than 1.

$50 \times 0.75^{n-1} < 1$

$(n - 1) \lg 0.75 < \lg(1 \div 50)$

$\lg 0.75$ is negative, so dividing reverses the inequality:

$n - 1 > \lg(1 \div 50) \div \lg 0.75 = 13.598$, so $n > 14.598$

The smallest integer n is 15

Answer: $n = 15$