Day 204 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Will it move? Limiting friction (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A box of mass 10 kg rests on a rough horizontal floor, with coefficient of friction 0.2. A horizontal force of 16 N is applied to the box.

Take $g = 9.8$ m s^{-2}.

Determine whether the box moves. Find the friction force, or the acceleration if it moves.

$R = 10 \times 9.8 = 98$, maximum friction $\mu R = 0.2 \times 98 = 19.6$

$16 < 19.6$ so the box stays at rest

Friction balances the applied force: $F = 16$

Answer: Does not move; friction = 16 N

2 Inverse normal (All boards)

The length X cm of a metal rod is normal with mean 45, s.d. 1.5.

Find k (3 s.f.): (a) $P(X < k) = 0.1$,

(b) $P(X > k) = 0.01$.

(a) $z = -1.282$, so $k = 45 + (-1.282) \times 1.5 = 43.1$

(b) $P(X < k) = 0.99$, $z = 2.326$, so $k = 45 + 2.326 \times 1.5 = 48.5$

Answer: (a) 43.1 (b) 48.5

3 Equation of a line through two points (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The points A and B have coordinates (7, 4) and (3, −5).

(a) Find the midpoint of AB.

(b) Find the line AB as $ax + by + c = 0$, with integer a, b, c.

(a) Midpoint $= \left(\frac{7+3}{2}, \frac{4+(-5)}{2}\right) = \left(5, -\frac{1}{2}\right)$

(b) Gradient $= \frac{-9}{-4} = \frac{9}{4}$

$y - 4 = \frac{9}{4}(x - 7)$

Multiply out and collect terms: $9x - 4y - 47 = 0$

Answer: (a) $\left(5, -\frac{1}{2}\right)$ (b) $9x - 4y - 47 = 0$

4 Segment of a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The shaded region is the segment cut off by the chord AB of a circle with centre O and radius 7 cm. Angle AOB = 1.3 radians.

Find the area of the segment, to 3 s.f.

Sector area $= \frac{1}{2} \times 7^2 \times 1.3 = 31.85$

Triangle area $= \frac{1}{2} \times 7^2 \times \sin 1.3 = 23.61$

Segment = sector − triangle $= 8.243$ cm^2

Answer: 8.24 cm^2

5 Transformations from a graph (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the curve $y = f(x)$. A and B are the turning points. Find the turning points of:

(a) $y = 2f(x)$ (b) $y = f(x - 1)$

(a) $y = 2f(x)$: A(−2, −2) maps to (−2, −4), B(4, 2) maps to (4, 4)

(b) $y = f(x - 1)$: A(−2, −2) maps to (−1, −2), B(4, 2) maps to (5, 2)

Answer: (a) (−2, −4), (4, 4) (b) (−1, −2), (5, 2)

6 Velocity parallel to a direction (AQA, Edexcel, OCR, MEI, Eduqas)

A particle moves so that at time t s its velocity is $\mathbf{v} = (2t^2 + 3)\mathbf{i} + (2t - 6)\mathbf{j}$ m s^{-1}.
Find the time when the particle is moving in the direction of $\mathbf{i}$, and its speed then.

Parallel to $\mathbf{i}$: $\mathbf{j}$ component = 0
$2t - 6 = 0$, so $t = 3$
$\mathbf{v} = (2 \times 3^2 + 3)\mathbf{i} = 21\mathbf{i}$
Answer: t = 3 s, speed 21 m s^{-1}

7 Discriminant problems (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The equation $5x^2 + kx + 45 = 0$ has equal roots.
Find the possible values of k.

Equal roots: $b^2 - 4ac = 0$
$k^2 - 4 \times 5 \times 45 = 0$
$k^2 = 900$
$k = \pm 30$
Answer: k = ±30

8 Binomial two-tailed test (All boards)

A factory says a proportion 0.5 of its parts are defective. In a sample of 30 parts, 11 are defective.
Test, at the 5% significance level, whether the proportion is different from 0.5.

H_0: $p = 0.5$, H_1: $p \neq 0.5$. Under H_0, $X \sim B(30, 0.5)$
$P(X \leq 11) = 0.1002$
Two-tailed: compare with 0.025. $0.1002 > 0.025$, so do not reject H_0
There is insufficient evidence that the proportion is different from 0.5
Answer: do not reject H_0: insufficient evidence that the proportion is different from 0.5

9 Optimisation: open box and tank (All boards)

Squares of side x cm are cut from the corners of a square card of side 36 cm and the sides are folded up to make an open box.
Find x for the greatest volume, and that volume.

$V = x(36 - 2x)^2 = 4x^3 - 144x^2 + 1296x$
$\frac{dV}{dx} = 12x^2 - 288x + 1296 = (6x - 36)(2x - 36) = 0$
$x < 18$, so $x = 6$
$\frac{d^2V}{dx^2} = 24x - 288 = -144 < 0$ at $x = 6$, so maximum
$V = 6 \times 24^2 = 3456$
Answer: x = 6, V = 3456 cm^3

10 Collinear points and parallel vectors (All boards)

Points A, B and C have position vectors $\begin{pmatrix} -3 \\ -3 \\ 3 \end{pmatrix}$, $\begin{pmatrix} -4 \\ 0 \\ 4 \end{pmatrix}$ and $\begin{pmatrix} p \\ 3 \\ 5 \end{pmatrix}$ respectively.
Given that A, B, C are collinear, find p.

$\mathrm{AB} = \begin{pmatrix} -1 \\ 3 \\ 1 \end{pmatrix}$, $\mathrm{AC} = \begin{pmatrix} p - -3 \\ 6 \\ 2 \end{pmatrix}$
$\mathrm{AC} = \lambda\mathrm{AB}$: component 2 gives $6 = \lambda \times 3$, so $\lambda = 2$
$p - -3 = 2 \times -1$, so $p = -5$
Answer: p = −5