Day 203 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Trig equations in radians (All boards)

Solve $\tan x = 3$ for $0 \leq x \leq 2\pi$,
giving answers to 3 significant figures.

$\tan \theta = 3$ for $0 \leq x \leq 2\pi$
Calculator: $x = 1.249$, then use the symmetry of the graph
$x = 1.249, 4.391$
Answer: x = 1.25, 4.39

2 Constant acceleration (suvat) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A cyclist moves in a straight line with constant acceleration. Its initial speed is 11 m s^{-1}, its final speed is 23.5 m s^{-1} and the motion lasts 5 s.
Find the distance travelled.

$s = \frac{1}{2}(u + v)t$
$s = \frac{1}{2} \times (11 + 23.5) \times 5$
Answer: 86.25 m

3 Integrating composite functions (reverse chain rule) (All boards)

Find (a) $\int \frac{6}{4x + 4}\,dx$,

(b) the exact value of $\int_0^2 \frac{6}{4x + 4}\,dx$.

(a) Reverse the chain rule and divide by the coefficient of x: $\frac{3\ln(4x + 4)}{2} + c$

(b) $\left[\frac{3\ln(4x + 4)}{2}\right]_0^2$
$= \frac{3\ln 3}{2}$

Answer: (a) $\frac{3\ln(4x + 4)}{2} + c$ (b) $\frac{3\ln 3}{2}$

4 Constant acceleration, two stages of working (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moving in a straight line with constant acceleration increases its speed from 2 m s^{-1} to 18 m s^{-1} while covering 40 m.
Find (a) the acceleration, (b) the time taken.

(a) $v^2 = u^2 + 2as$: $324 = 4 + 80\,a$, so $a = 4$
(b) $v = u + at$: $18 = 2 + 4\,t$
$t = 16 \div 4 = 4$
Answer: (a) 4 m s^{-2} (b) 4 s

5 Poisson: changing the interval (CIE)

Faults occur at random in a cable at an average rate of 3 per hour of operation.
Find the probability that in 15 minutes there are exactly 0 (3 s.f.).

Rate in 15 minutes: $\lambda = 3 \times \frac{15}{60} = 0.75$
$X \sim \text{Po}(0.75)$
$P(X = 0) = e^{-0.75} \times \frac{0.75^0}{0!} = 0.472$
Answer: 0.472

6 R cos(θ − α) form: express and maximum (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Express $6\cos\theta + 8\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < 90°$. Give α to 1 d.p.

$R\cos\alpha = 6$, $R\sin\alpha = 8$
$R = \sqrt{6^2 + 8^2} = 10$
$\tan\alpha = \frac{8}{6}$, so $\alpha = 53.1°$
Answer: $10\cos(\theta - 53.1°)$

7 Partial fractions (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Express $\frac{-7x-12}{(x+4)(x-4)}$ in partial fractions.

$\frac{-7x-12}{(x+4)(x-4)} \equiv \frac{A}{(x+4)} + \frac{B}{(x-4)}$, so $-7x-12 \equiv A(x-4) + B(x+4)$

$x = -4$: $16 = A \times (-8)$, so $A = -2$

$x = 4$: $-40 = B \times 8$, so $B = -5$

Answer: $\mathbf{-\frac{2}{(x+4)} - \frac{5}{(x-4)}}$

8 Hypothesis test for a normal mean (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Packet masses are normal, s.d. 5 g. Claimed mean 50 g; 16 packets have mean 51.7 g.
Test at the 5% level whether the true mean is greater than 50.

$H_0: \mu = 50$, $H_1: \mu > 50$. Under H_0, $\overline{X} \sim N(50, \frac{5^2}{16})$

$z = \frac{51.7 - 50}{5/\sqrt{16}} = 1.360$

$P(Z > 1.360) = 0.0869 > 0.05$

Do not reject H_0: there is insufficient evidence that the mean has increased

Answer: do not reject H_0: insufficient evidence of an increase in the mean

9 Solving index equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $27^{2x-4} = 3^{-3x+3}$.

Write both sides as powers of 3: $3^{3(2x-4)} = 3^{-3x+3}$

Equate the indices: $6x - 12 = -3x + 3$

$9x = 15$

$x = \frac{5}{3}$

Answer: $\mathbf{x = \frac{5}{3}}$

10 Binomial expansion (any power) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Expand $(1-2x)^{-1}$ in ascending powers of x up to and including the term in x^3.

(b) State the range of values of x for which the expansion is valid.

$(1+ax)^n = 1 + n(ax) + \frac{n(n-1)}{2!}(ax)^2 + \frac{n(n-1)(n-2)}{3!}(ax)^3 + \ldots$

Here $n = -1$ and $a = -2$:

coefficients: x: 2, x^2: 4, x^3: 8

Valid when $|2x| < 1$, that is $|x| < \frac{1}{2}$

Answer: (a) $1 + 2x + 4x^2 + 8x^3 + \ldots$ (b) $|x| < \frac{1}{2}$