**Day 202** **A Level Maths: B Tier 10-a-day: Model Solutions**

## 1 Rates of change (connected rates) (All boards)

The edge of a cube increases at 0.3 cm per second.
Find the rate of increase of (a) its volume, (b) its surface area, when the edge is 6 cm.

$V = x^3$, so $\frac{dV}{dt} = 3x^2\frac{dx}{dt} = 3 \times 6^2 \times 0.3 = \frac{162}{5}$

$S = 6x^2$, so $\frac{dS}{dt} = 12x\frac{dx}{dt} = 12 \times 6 \times 0.3 = \frac{108}{5}$

**Answer: (a) $\frac{162}{5}$ cm$^3$/s (b) $\frac{108}{5}$ cm$^2$/s**

## 2 Newton's second law: lift and tension (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A lift of mass 200 kg is raised and lowered by a vertical cable. The lift accelerates upwards at 1 m s$^{-2}$.
Take g = 9.8 m s$^{-2}$.
Find the tension in the cable.

Resolve vertically (up positive): $T - 200 \times 9.8 = 200 \times 1$
$T = 200(9.8 + 1)$
**Answer: 2160 N**

## 3 Comparing box plots (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The box plots show the times taken (minutes) to complete a puzzle for two groups.
(a) Find the interquartile range of each group.
(b) Compare the two distributions.

(a) Class A: 60 − 30 = 30. Class B: 40 − 20 = 20
(b) Medians: 45 (Class A) and 25 (Class B), so Class A has the higher median
IQRs: Class A has the larger IQR, so its scores are more spread out
**Answer: (a) 30 and 20 (b) Class A has the higher median; Class A is more spread out (larger IQR), Class B more consistent**

## 4 Inverse trigonometric functions (AQA, Edexcel, OCR, MEI, Eduqas)

Find the exact value of $\arcsin(\sin -\frac{\pi}{4})$.

$\sin -\frac{\pi}{4} = -\frac{\sqrt{2}}{2}$

$\arcsin(-\frac{\sqrt{2}}{2})$ must lie in $[-\frac{\pi}{2}, \frac{\pi}{2}]$, so the value is $-\frac{\pi}{4}$

**Answer: $-\frac{\pi}{4}$**

## 5 Scatter diagram (AQA, Edexcel, OCR, MEI, Eduqas)

The scatter diagram shows eight pairs of values of x and y.
(a) Describe the correlation.
(b) Why should y = 6 + 2x not be used for x = 20?

(a) The points lie close to a straight line rising: strong positive correlation
(b) The data cover x = 1 to 10; x = 20 is outside this range
so using the line would be extrapolation and may be unreliable
**Answer: (a) strong positive correlation (b) extrapolation outside the range of the data**

## 6 Combined inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the set of values of x for which
$3x + 2 \geq 0$ and $x^2 - 9 \geq 0$.

$3x + 2 \geq 0$ gives $x \geq -\frac{2}{3}$

$x^2 - 9 \geq 0$: $(x + 3)(x - 3) \geq 0$ gives $x \leq -3$ or $x \geq 3$
Both must hold: $x \geq 3$
**Answer: $x \geq 3$**

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## 7 Instantaneous rest and displacement (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle P moves on a straight line from O. At time t s its velocity is $6t^2 - 42t + 72$ m s$^{-1}$.
Find the first time P is at instantaneous rest, and its distance from O then.

$v = 0$: $6(t - 3)(t - 4) = 0$, so $t = 3$ or $t = 4$

First time $t = 3$

$s = \int_0^3 v\,dt = [\,2t^3 - 21t^2 + 72t\,]$ from 0 to 3

**Answer: t = 3 s; 81 m**

## 8 Arc length and sector area (radians) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A sector of a circle has radius 6 cm and arc length 7.2 cm.
Find (a) the angle of the sector in radians, (b) the area of the sector.

$s = r\theta$, so $\theta = \frac{7.2}{6} = 1.2$

Area $= \frac{1}{2}r^2\theta = \frac{1}{2} \times 6^2 \times 1.2 = 21.6$

**Answer: (a) 1.2 rad (b) 21.6 cm$^2$**

## 9 Domain and range (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = \sqrt{-4x + 2} + 4$.
(a) Find the largest possible domain of f.
(b) Find the range of f.

The expression under the root must be non-negative: $-4x + 2 \geq 0$

$-4x \geq -2$ gives $x \leq \frac{1}{2}$

$\sqrt{-4x + 2} \geq 0$, so $f(x) \geq 4$

**Answer: (a) $x \leq \frac{1}{2}$ (b) $f(x) \geq 4$**

## 10 Rationalising the denominator (AQA, Edexcel, OCR, MEI, Eduqas)

Express $\frac{2 + 4\sqrt{7}}{3 + \sqrt{7}}$ in the form $p + q\sqrt{7}$, where p and q are integers.

Multiply top and bottom by the conjugate $(3 - \sqrt{7})$:
Denominator: $(3 + \sqrt{7})(3 - \sqrt{7}) = 3^2 - 1^2 \times 7 = 2$
Numerator: $(2 + 4\sqrt{7})(3 - \sqrt{7}) = -22 + 10\sqrt{7}$
So the fraction $= \frac{-22 + 10\sqrt{7}}{2} = -11 + 5\sqrt{7}$

**Answer: $-11 + 5\sqrt{7}$**