Day 201 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Quadratic inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $-x^2 \geq -4$.

Rearrange: $-x^2 + 4 \geq 0$
Factorise: $-(x - 2)(x + 2) \geq 0$
Critical values: $x = -2$ and $x = 2$
Sketch the $\cap$-shaped parabola: $-2 \leq x \leq 2$
Answer: $-2 \leq x \leq 2$

2 Car towing a trailer (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A car of mass 1500 kg tows a trailer of mass 200 kg along a straight horizontal road by a light horizontal tow bar. The resistances are 400 N on the car and 250 N on the trailer.
The engine's driving force is 1500 N. Find the acceleration and the tension in the tow bar.

Whole system: $1500 - 400 - 250 = 1700\ a$
$a = 0.5$
Trailer: $T - 250 = 200 \times 0.5$
Answer: $a = 0.5$ m s^{-2}; $T = 350$ N

3 Interpreting regression (AQA, Edexcel, OCR, MEI, Eduqas)

Regression line (x from 2 to 10): $y = 3.2 + 0.8x$.
x is the advertising spend, y the weekly sales.
(a) Interpret the gradient.
(b) Estimate y when x = 15 and comment.

(a) The gradient: for each extra £100 of advertising, y is predicted to increase by 0.8
(b) $y = 3.2 + (0.8) \times 15 = 15.2$
x = 15 lies outside the range of the data (extrapolation), so the estimate may be unreliable
Answer: (a) y increases by 0.8 per extra £100 of advertising (b) 15.2, unreliable (extrapolation)

4 Inequalities with exponents and logs (All boards)

Solve $2^x > 201$, giving your answer to 3 s.f.

Take logs: $x \ln 2 > \ln 201$
$x > \frac{\ln 201}{\ln 2} = 7.651$
Answer: $x > 7.65$

5 Coefficients in a binomial expansion (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the term independent of x in the expansion of

$(x^2 + \frac{3}{x})^9$.

The general term is $C(9, k)(x^2)^{9-k}(3/x)^k$, with power of x equal to $18 - 3k$
Independent of x: $18 - 3k = 0$, so $k = 6$
Term $= 84 \times 3^6 = 61236$
Answer: 61236

6 Double angle equations (All boards)

Solve $3 \sin 2x = 2 \cos x$ for $0 \leq x \leq 360°$,
giving answers to 1 d.p. where necessary.

Use the double angle formula: $6 \sin x \cos x - 2 \cos x = 0$
$2 \cos x(3 \sin x - 1) = 0$
$\cos x = 0$: $x = 90.0°, 270.0°$
$\sin x = \frac{1}{3}$: $x = 19.5°, 160.5°$
Answer: $x = 19.5°, 90.0°, 160.5°, 270.0°$

7 Transformations from a graph (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the curve y = f(x). A and B are the turning points. Find the turning points of:

(a) $y = f(\frac{x}{2})$ (b) $y = f(x) - 2$

(a) $y = f(\frac{x}{2})$: A(0, −1) maps to (0, −1), B(6, 5) maps to (12, 5)

(b) $y = f(x) - 2$: A(0, −1) maps to (0, −3), B(6, 5) maps to (6, 3)

Answer: (a) (0, −1), (12, 5) (b) (0, −3), (6, 3)

8 Inverse normal (All boards)

A journey takes X minutes, normal with mean 40, s.d. 0.5.
Find k (3 s.f.): (a) P(X < k) = 0.1,
(b) P(X > k) = 0.1.

(a) z = −1.282, so k = 40 + (−1.282) × 0.5 = 39.4
(b) P(X < k) = 0.9, z = 1.282, so k = 40 + 1.282 × 0.5 = 40.6

Answer: (a) 39.4 (b) 40.6

9 Partial fractions and integration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Write $\frac{-5x - 16}{(x + 2)(x + 4)}$ in partial fractions.

(b) Hence evaluate $\int_0^4 \frac{-5x - 16}{(x + 2)(x + 4)}\,dx$,
giving the answer as a single logarithm.

(a) $\frac{-5x - 16}{(x + 2)(x + 4)} \equiv \frac{A}{x + 2} + \frac{B}{x + 4}$, so $-5x - 16 \equiv A(x + 4) + B(x + 2)$
x = −2: A = −3; x = −4: B = −2

(b) $\int_0^4 (\frac{-3}{x + 2} + \frac{-2}{x + 4})\,dx = [-3\ln(x + 2) - 2\ln(x + 4)]_0^4$

$= -3\ln\frac{6}{2} - 2\ln\frac{8}{4} = \ln(\frac{1}{108})$

Answer: (a) A = −3, B = −2 (b) $\ln(\frac{1}{108})$

10 Pulling at an angle: constant speed on rough surface (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A suitcase of mass 20 kg is dragged at constant speed along rough horizontal ground by a rope inclined at angle α above the horizontal, where $\cos\alpha = \frac{12}{13}$.

The coefficient of friction is 0.2. Take g = 9.8 m s^{-2}. Find the tension in the rope.

Vertical: $R + T\sin\alpha = mg$, so $R = 196 - \frac{5}{13}T$

Horizontal: $T\cos\alpha = \mu R$: $\frac{12}{13}T = 0.2(196 - \frac{5}{13}T)$

$T(\cos\alpha + \mu\sin\alpha) = \mu mg$: $T \times 1 = 0.2 \times 20 \times 9.8$ (since $\cos\alpha + \mu\sin\alpha = 1$)

Answer: 39.2 N