Day 200 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Constant acceleration with vectors (AQA, Edexcel, OCR, MEI, Eduqas)

A particle has velocity $(3\mathbf{i} - \mathbf{j})$ m s^{-1} at time t = 0 and moves with constant acceleration $(-3\mathbf{i} - 3\mathbf{j})$ m s^{-2}.
Find its velocity and speed when t = 5.

$\mathbf{v} = \mathbf{u} + \mathbf{a}t = (3\mathbf{i} - \mathbf{j}) + 5(-3\mathbf{i} - 3\mathbf{j})$
$= -12\mathbf{i} - 16\mathbf{j}$
Speed $= \sqrt{(-12^2 + -16^2)}$
Answer: $\mathbf{v} = (-12\mathbf{i} - 16\mathbf{j})$ m s^{-1}; speed 20 m s^{-1}

2 Segment of a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The shaded region is the segment cut off by the chord AB of a circle with centre O and radius 11 cm. Angle AOB = 1.1 radians.
Find the area of the segment, to 3 s.f.

Sector area $= \frac{1}{2} \times 11^2 \times 1.1 = 66.55$

Triangle area $= \frac{1}{2} \times 11^2 \times \sin 1.1 = 53.92$

Segment = sector − triangle = 12.63 cm^2
Answer: 12.6 cm^2

3 Domain and range (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = \sqrt{x - 1} + 3$.
(a) Find the largest possible domain of f.
(b) Find the range of f.

The expression under the root must be non-negative: $x - 1 \geq 0$
$x \geq 1$ gives $x \geq 1$
$\sqrt{x - 1} \geq 0$, so $f(x) \geq 3$
Answer: (a) $x \geq 1$ (b) $f(x) \geq 3$

4 Probability without replacement (All boards)

A bag has 6 white and 5 black sweets; two are taken without replacement.
(a) Find P(same colour).
(b) Given they are the same colour, find P(both white).

(a) P(white white) $= \frac{6}{11} \times \frac{5}{10} = \frac{3}{11}$

P(black black) $= \frac{5}{11} \times \frac{4}{10} = \frac{2}{11}$

P(same) $= \frac{3}{11} + \frac{2}{11} = \frac{5}{11}$

(b) P $= \frac{3}{11} \div \frac{5}{11} = \frac{3}{5}$

Answer: (a) $\frac{5}{11}$ (b) $\frac{3}{5}$

5 Double angle values from a given ratio (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Given that $\cos\theta = -\frac{5}{13}$ and $90° < \theta < 180°$,
find the exact value of $\tan 2\theta$.

$\sin\theta = \sqrt{1 - (-\frac{5}{13})^2} = \frac{12}{13}$

$\sin 2\theta = -\frac{120}{169}$ and $\cos 2\theta = -\frac{119}{169}$

$\tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta} = \frac{120}{119}$

Answer: $\frac{120}{119}$

6 Completing the square proof (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Prove that $-x^2 + 8x - 23 < 0$ for all real values of x.

Complete the square: $-x^2 + 8x - 23 = -(x - 4)^2 - 7$

$(x - 4)^2 \geq 0$ for all real x, so $-(x - 4)^2 \leq 0$.

Adding -7: the expression is ≤ -7, so < 0 for all real x.

Answer: $-x^2 + 8x - 23 < 0$ for all x

7 Reduction to linear form (logarithms) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The variables x and y satisfy $y = Ae^{kx}$. A graph of ln y against x is a straight line through (0, 1) and (4, 2.6). Find A to 3 s.f. and k.

$\ln y = \ln A + kx$, so $\ln A = 1$ and $A = e^1 = 2.718$

$k = \text{gradient} = \frac{2.6 - 1}{4 - 0} = 0.4$

Answer: A = 2.72, k = 0.4

8 Binomial two-tailed test (All boards)

A factory says a proportion 0.3 of its parts are defective. In a sample of 25 parts, 3 are defective. Test, at the 5% significance level, whether the proportion is different from 0.3.

H_0: $p = 0.3$, H_1: $p \neq 0.3$. Under H_0, $X \sim B(25, 0.3)$

$P(X \leq 3) = 0.0332$

Two-tailed: compare with 0.025. $0.0332 > 0.025$, so do not reject H_0

There is insufficient evidence that the proportion is different from 0.3

Answer: do not reject H_0: insufficient evidence that the proportion is different from 0.3

9 Inclined plane: force to hold or move the particle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A block of mass 5 kg is held on a rough plane ($\tan \alpha = \frac{4}{3}$, $\mu = 0.2$) by a force P N up the plane, as shown.

Take $g = 9.8$ m s^{-2}. The block is about to move up the plane. Find P.

$R = mg \cos \alpha = 49 \times 3/5 = 29.4$

Friction (limiting) $F = \mu R = 5.88$

Moving up, friction acts down: $P = mg \sin \alpha + F$

$mg \sin \alpha = 39.2$

Answer: 45.08 N

10 Parametric curve meeting a line (AQA, Edexcel, OCR, MEI, Eduqas)

A curve has parametric equations $x = 2t$, $y = t^2 - t$.
The line $y = x$ meets the curve at two points.
Find the coordinates of these points.

Substitute x and y into the line: $t^2 - t = 2t$

$t^2 - 3t = 0$, so $(t - 3)t = 0$

$t = 0$ or $t = 3$, giving the points (0, 0) and (6, 6)

Answer: (0, 0) and (6, 6)