Day 20 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Equation of a line through two points (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The points A and B have coordinates $(-2, -6)$ and $(-4, 3)$.
(a) Find the midpoint of AB.
(b) Find the line AB as $ax + by + c = 0$, with integer a, b, c.

(a) Midpoint $= \left(\frac{-2 + (-4)}{2}, \frac{-6 + 3}{2}\right) = \left(-3, -\frac{3}{2}\right)$

(b) Gradient $= \frac{9}{-2} = -\frac{9}{2}$

$y + 6 = -\frac{9}{2}(x + 2)$

Multiply out and collect terms: $9x + 2y + 30 = 0$

Answer: (a) $\left(-3, -\frac{3}{2}\right)$ (b) $9x + 2y + 30 = 0$

2 Union, conditional probability and independence (All boards)

P(A) = 0.4, P(B) = 0.3, P(A ∩ B) = 0.12.
Find (a) P(A ∪ B), (b) P(A | B).
(c) Are A and B independent?

(a) $P(A \cup B) = 0.4 + 0.3 - 0.12 = 0.58$

(b) $P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{0.12}{0.3} = 0.4$

(c) $P(A)P(B) = 0.4 \times 0.3 = 0.12$, which equals $P(A \cap B)$

Answer: (a) 0.58 (b) 0.4 (c) independent

3 Power, driving force and resistance (CIE)

A car of mass 1000 kg moves along a horizontal road. The engine works at a constant rate of 48 kW. The resistance to motion is constant at 1000 N.
Find the acceleration when the speed is 15 m s^{-1}.

$P = Fv$: $F = 48000 \div 15 = 3200$ N

$F - R = ma$: $3200 - 1000 = 1000\,a$

Answer: 2.2 m s^{-2}

4 Comparing box plots (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The box plots show the times taken (minutes) to complete a puzzle for two groups.
(a) Find the interquartile range of each group.
(b) Compare the two distributions.

(a) Class A: 50 − 35 = 15. Class B: 40 − 30 = 10

(b) Medians: 40 (Class A) and 35 (Class B), so Class A has the higher median

IQRs: Class A has the larger IQR, so its scores are more spread out

Answer: (a) 15 and 10 (b) Class A has the higher median; Class A is more spread out (larger IQR), Class B more consistent

5 Collinear points and parallel vectors (All boards)

Points A, B and C have position vectors $\begin{pmatrix} 1 \\ -2 \\ -3 \end{pmatrix}$, $\begin{pmatrix} 2 \\ 0 \\ -5 \end{pmatrix}$ and $\begin{pmatrix} p \\ 2 \\ -7 \end{pmatrix}$ respectively.
Given that A, B, C are collinear, find p.

$\overrightarrow{AB} = \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$, $\overrightarrow{AC} = \begin{pmatrix} p - 1 \\ 4 \\ -4 \end{pmatrix}$

$\overrightarrow{AC} = \lambda \overrightarrow{AB}$: component 2 gives $4 = \lambda \times 2$, so $\lambda = 2$

$p - 1 = 2 \times 1$, so $p = 3$

Answer: $p = 3$

6 Combined inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the set of values of x for which
$5x + 4 > -3$ and $x^2 - x \geq 0$.

$5x + 4 > -3$ gives $x > -\frac{7}{5}$

$x^2 - x \geq 0$: $(x - 1)x \geq 0$ gives $x \leq 0$ or $x \geq 1$

Both must hold: $-\frac{7}{5} < x \leq 0$ or $x \geq 1$

Answer: $-\frac{7}{5} < x \leq 0$ or $x \geq 1$

7 Fixed-point iteration (All boards)

(a) Show that $x^3 - 3x - 7 = 0$ can be written as $x = \sqrt[3]{3x + 7}$.
(b) Use $x_{n+1} = \sqrt[3]{3x_n + 7}$ with $x_0 = 2$ to find x_1, x_2, x_3 to 4 d.p.

(a) $x^3 = 3x + 7$, so $x = \sqrt[3]{3x + 7}$
(b) $x_1 = 2.3513$
$x_2 = 2.4132$
$x_3 = 2.4238$
Answer: $x_1 = 2.3513$, $x_2 = 2.4132$, $x_3 = 2.4238$

8 Binomial expansion (any power) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Expand $(8 - 4x)^{\frac{1}{3}}$ in ascending powers of x up to and including the term in x^3.
(b) State the range of values of x for which the expansion is valid.

Take out 8: $(8 - 4x)^{\frac{1}{3}} = 8^{\frac{1}{3}}(1 + -\frac{1}{2}x)^{\frac{1}{3}} = 2(1 + -\frac{1}{2}x)^{\frac{1}{3}}$

Expand $(1 + -\frac{1}{2}x)^{\frac{1}{3}} = 1 + n(ax) + \frac{n(n-1)}{2!}(ax)^2 + \frac{n(n-1)(n-2)}{3!}(ax)^3 + ...$

Multiplying by 2: coefficients of 1, x, x^2, x^3 are 2, $-\frac{1}{3}$, $-\frac{1}{18}$, $-\frac{5}{324}$

Valid when $|\frac{1}{2}x| < 1$, that is $|x| < 2$

Answer: (a) $2 - \frac{1}{3}x - \frac{1}{18}x^2 - \frac{5}{324}x^3 + ...$ (b) $|x| < 2$

9 Inclined plane: force to hold or move the particle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A block of mass 5 kg is held on a rough plane ($\tan\alpha = \frac{4}{3}$, $\mu = 0.5$) by a force P N up the plane, as shown.

Take $g = 9.8$ m s^{-2}. The block is about to move down the plane. Find P.

$R = mg\cos\alpha = 49 \times 3/5 = 29.4$
Friction (limiting) $F = \mu R = 14.7$
Moving down, friction acts up: $P + F = mg\sin\alpha$
$mg\sin\alpha = 39.2$
Answer: 24.5 N

10 Position vectors and dividing a line in a ratio (All boards)

A has position vector $\begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}$ and B has position vector $\begin{pmatrix} 1 \\ 7 \\ -1 \end{pmatrix}$.

The point P lies on AB with AP : PB = 3 : 1.

Find the position vector of P.

$\text{AB} = \begin{pmatrix} 0 \\ 8 \\ 0 \end{pmatrix}$

$\text{AP} = \frac{3}{4}\text{AB} = \begin{pmatrix} 0 \\ 6 \\ 0 \end{pmatrix}$

$\text{OP} = \text{OA} + \text{AP} = \begin{pmatrix} 1 \\ 5 \\ -1 \end{pmatrix}$

Answer: $\begin{pmatrix} \mathbf{1} \\ \mathbf{5} \\ \mathbf{-1} \end{pmatrix}$