Day 2 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Trig equations in radians (All boards)

Solve $\sin(x + 0.3) = 0.4$ for $0 \leq x \leq 2\pi$,
giving answers to 3 significant figures.

Let $\theta = x + 0.3$, so $\sin\theta = 0.4$ for $0.3 \leq \theta \leq 2\pi + 0.3$
Calculator: $\theta = 0.4115$, then use the symmetry of the graph
$\theta = 0.4115, 2.730$
$x = \theta - 0.3$
$x = 0.1115, 2.430$
Answer: x = 0.112, 2.43

2 Combined inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the set of values of x for which
$2x + 1 > -8$ and $x^2 + 2x \geq 0$.

$2x + 1 > -8$ gives $x > -\frac{9}{2}$

$x^2 + 2x \geq 0$: $x(x + 2) \geq 0$ gives $x \leq -2$ or $x \geq 0$

Both must hold: $-\frac{9}{2} < x \leq -2$ or $x \geq 0$

Answer: $-\frac{9}{2} < x \leq -2$ or $x \geq 0$

3 Integration by substitution (given substitution) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Use the substitution $u = x^2 + 1$ to find the exact value of

$\int_0^2 x(x^2 + 1)^2\,dx$.

$\frac{du}{dx} = 2x$, so $x\,dx = \frac{1}{2}\,du$
Limits: $x = 0$ gives $u = 1$, $x = 2$ gives $u = 5$
$\int_1^5 \frac{1}{2}u^2\,du = \left[\frac{u^3}{6}\right]_1^5$
$= \frac{62}{3}$

Answer: $\frac{62}{3}$

4 Comparing box plots (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The box plots show the times taken (minutes) to complete a puzzle for two groups.
(a) Find the interquartile range of each group.
(b) Compare the two distributions.

(a) Year 12: $50 - 40 = 10$. Year 13: $35 - 15 = 20$
(b) Medians: 45 (Year 12) and 20 (Year 13), so Year 12 has the higher median
IQRs: Year 13 has the larger IQR, so its scores are more spread out
Answer: (a) 10 and 20 (b) Year 12 has the higher median; Year 13 is more spread out (larger IQR), Year 12 more consisten

5 Lift: weighing scales reading (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A woman of mass 40 kg stands on weighing scales in a lift. The lift decelerates at 0.5 m s^{-2} while moving upwards.
Take g = 9.8 m s^{-2}.
Find the force exerted on the woman by the scales.

The acceleration is downwards, magnitude 0.5
Taking up as positive: $R - 40 \times 9.8 = -40 \times 0.5$
$R = 40(9.8 - 0.5)$
Answer: 372 N

6 Graph of the modulus of a quadratic (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the curve y = f(x), a quadratic.
(a) Find the coordinates of the local maximum point of y = |f(x)|.
(b) How many solutions has |f(x)| = 16?

(a) y = |f(x)| reflects the part of the curve below the x-axis in the x-axis:
(1, −16) becomes (1, 16)
(b) The maximum of the middle part is 16. Compare k = 16 with 16:
k = 16 gives 3 intersections with y = k.
Answer: (a) (1, 16) (b) 3

7 Integrating velocity (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves in a straight line. At time t seconds its velocity is $v = 3t^2 - 4t$ m s^{-1}.
Find the distance moved between t = 2 and t = 6. (The particle does not change direction.)

$s = \int_2^6 (3t^2 - 4t)\,dt$

$= [\,t^3 - 2t^2\,]$ from t = 2 to t = 6
$= 144 - 0$
Answer: 144 m

8 Tangent lines of a given gradient (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line 4x + 3y = k is a tangent to the circle
$(x + 3)^2 + (y - 2)^2 = 100$.
Find the two possible values of k.

The radius to the point of contact is perpendicular to the line, so it is parallel to (4, 3).
A unit step in this direction is (4, 3)/5, so the points of contact are
(−3, 2) ± 10 × (4, 3)/5 = (−3, 2) ± (8, 6)
T = (5, 8) or (−11, −4), and k = 4x + 3y gives:
k = 44 or k = −56
Answer: k = 44 or k = −56

9 Optimisation: open box and tank (All boards)

A closed cylinder has radius r cm and volume 128π cm^3.
Show that its surface area is $S = 2\pi r^2 + \frac{256\pi}{r}$, and find the minimum S.

Height $h = \frac{128}{r^2}$, so $S = 2\pi r^2 + 2\pi rh = 2\pi r^2 + \frac{256\pi}{r}$

$\frac{dS}{dr} = 4\pi r - \frac{256\pi}{r^2} = 0$, so $r^3 = 64$ and r = 4

$\frac{d^2S}{dr^2} = 4\pi + \frac{512\pi}{r^3} > 0$, so minimum

$S = 2\pi \times 16 + \frac{256\pi}{4} = 96\pi$ cm^2

Answer: $S = 96\pi$ cm^2

10 Binomial two-tailed test (All boards)

A coin is thought to land heads with probability 0.4. It is tossed 50 times and lands heads 15 times.
Test, at the 5% significance level, whether the probability of heads is different from 0.4.

H_0: p = 0.4, H_1: $p \neq 0.4$. Under H_0, X ~ B(50, 0.4)
$P(X \leq 15) = 0.0955$
Two-tailed: compare with 0.025. 0.0955 > 0.025, so do not reject H_0
There is insufficient evidence that the probability of heads is different from 0.4
Answer: do not reject H_0: insufficient evidence that the probability of heads is different from 0.4