Day 199 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Describing transformations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Describe the single transformation that maps the graph of y = f(x) onto the graph of:
(a) y = f(2x)
(b) y = f(x) + 6
(c) y = 2f(x)

(a) y = f(2x) is a stretch, scale factor $\frac{1}{2}$, parallel to the x-axis.

(b) y = f(x) + 6 is a translation $\begin{pmatrix} 0 \\ 6 \end{pmatrix}$.

(c) y = 2f(x) is a stretch, scale factor 2, parallel to the y-axis.

Answer: (a) stretch, scale factor $\frac{1}{2}$, parallel to the x-axis (b) translation $\begin{pmatrix} 0 \\ 6 \end{pmatrix}$ (c) stretch, scale factor 2, parallel to the y-axis

2 Constant acceleration (suvat) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A cyclist moves in a straight line with constant acceleration. Its initial speed is 10 m s^{-1}, its final speed is 30 m s^{-1} and it travels 80 m.
Find the time taken.

$s = \frac{1}{2}(u + v)t$
$80 = \frac{1}{2} \times (10 + 30) \times t$
$t = 160 \div 40$
Answer: 4 s

3 Composite functions (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = x + 3$ and $g(x) = x^2 + 3$ for $x \in \mathbb{R}$.
(a) Find fg(x).
(b) Solve gf(x) = 52.

$fg(x) = f(x^2 + 3) = (x^2 + 3) + 3 = x^2 + 6$
$gf(x) = (x + 3)^2 + 3 = 52$
$(x + 3)^2 = 49$, so $x + 3 = \pm 7$
$x = -10$ or $x = 4$
Answer: (a) $fg(x) = x^2 + 6$ (b) $x = -10$ or $x = 4$

4 Tangents and normals to exponential and log curves (All boards)

The curve C has equation $y = x \ln x - 2x$.
Find the equation of the normal to C at the point where x = 1, in the form y = mx + c.

$\frac{dy}{dx} = \ln x - 1$

At x = 1: $y = -2$ and $\frac{dy}{dx} = -1$
Normal gradient $= -1 \div (-1) = 1$
$y + 2 = 1(x - 1)$, so $y = x - 3$
Answer: $y = x - 3$

5 Differentiating displacement (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves on a straight line. Its displacement from O at time t seconds is $s = 4t^3 - t^2 - 6t$ metres.
Find its velocity and acceleration when t = 3.

$v = ds/dt = 12t^2 - 2t - 6$
$a = dv/dt = 24t - 2$
t = 3: v = 96, a = 70
Answer: $v = 96$ m s^{-1}, $a = 70$ m s^{-2}

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6 Integrals of the form f′(x)/f(x) (All boards)

Find the exact value of

$\int_0^2 \frac{4x + 6}{x^2 + 3x + 2}\,dx$,

giving your answer in the form a ln b.

The numerator is a multiple of the derivative of the denominator

$\int \frac{4x + 6}{x^2 + 3x + 2}\,dx = 2\ln(x^2 + 3x + 2)$

$= [2\ln(x^2 + 3x + 2)]_0^2$

$= \ln 36$

Answer: ln 36

7 Parametric curve meeting a line (AQA, Edexcel, OCR, MEI, Eduqas)

A curve has parametric equations $x = 3t + 1$, $y = 2t^2 + 4t + 2$.
The line $y = 2x + 4$ meets the curve at two points.
Find the coordinates of these points.

Substitute x and y into the line: $2t^2 + 4t + 2 = 6t + 6$

$2t^2 - 2t - 4 = 0$, so $2(t - 2)(t + 1) = 0$

$t = -1$ or $t = 2$, giving the points $(-2, 0)$ and $(7, 18)$

Answer: $(-2, 0)$ and $(7, 18)$

8 Double angle equations (All boards)

Solve $\sin 2x = \sin x$ for $0 \leq x \leq 360°$,
giving answers to 1 d.p. where necessary.

Use the double angle formula: $2\sin x \cos x - \sin x = 0$

$\sin x(2\cos x - 1) = 0$

$\sin x = 0$: $x = 0.0°, 180.0°, 360.0°$

$\cos x = \frac{1}{2}$: $x = 60.0°, 300.0°$

Answer: x = 0.0°, 60.0°, 180.0°, 300.0°, 360.0°

9 Distribution of the sample mean (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The time, X minutes, to complete a task is normally distributed with mean 150 and s.d. 5.
A random sample of 4 is taken. Find the probability that the mean time is more than 152 (3 s.f.).

$\overline{X} \sim N(150, \frac{5^2}{4})$, standard deviation 2.500

$z = \frac{152 - 150}{2.500} = 0.8000$

$P = 0.212$

Answer: 0.212

10 Hypothesis test for correlation (AQA, Edexcel, OCR, MEI, Eduqas)

20 pairs of height and arm span give $r = -0.515$.
Two-tailed critical value (5%): 0.4438.
Test for correlation.

$H_0: \rho = 0$, $H_1: \rho \neq 0$

$|r| = 0.515 > 0.4438$

r is in the critical region, so reject H_0

Conclusion: there is evidence of correlation between height and arm span

Answer: reject H_0: there is evidence of correlation between height and arm span