Day 198 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Stratified sampling (AQA, Edexcel, OCR, MEI, Eduqas)

A survey covers 56 urban, 32 suburban and 36 rural households.
A stratified sample of size 31 is taken. Find the number selected from each group.

Total = 56 + 32 + 36 = 124

Sampling fraction = 31/124 = $\frac{1}{4}$

urban: 56 ÷ 4 = 14
suburban: 32 ÷ 4 = 8
rural: 36 ÷ 4 = 9
Answer: urban 14, suburban 8, rural 9

2 Arc length and sector area (radians) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows a sector OAB of a circle with centre O, radius 15 cm and angle 1.4 radians.
Find (a) the arc length AB, (b) the area of the sector.

Arc length = $r\theta = 15 \times 1.4 = 21$ cm

Area = $\frac{1}{2}r^2\theta = \frac{1}{2} \times 15^2 \times 1.4 = 157.5$ cm^2

Answer: (a) 21 cm (b) 157.5 cm^2

3 Free fall from rest (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A ball is dropped from rest from a window, 30.625 m above the ground.
Take g = 9.8 m s^{-2}.
Find (a) the time taken to reach the ground, (b) its speed on landing.

(a) $s = ut + \frac{1}{2}at^2$: $30.625 = \frac{1}{2} \times 9.8 \times t^2$
$t^2 = 30.625 \div 4.9 = 6.25$, so $t = 2.5$
(b) $v = u + at = 9.8 \times 2.5$
Answer: (a) 2.5 s (b) 24.5 m s^{-1}

4 Completing the square proof (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Prove that $3x^2 - 24x + 51 > 0$ for all real values of x.

Complete the square: $3x^2 - 24x + 51 = 3(x - 4)^2 + 3$
$(x - 4)^2 \geq 0$ for all real x, so $3(x - 4)^2 \geq 0$.
Adding 3: the expression is ≥ 3, so > 0 for all real x.
Answer: $3x^2 - 24x + 51 > 0$ for all x

5 Reading a histogram (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The histogram shows the times taken by a group of people to complete a task.
Estimate the number of people who took between 20 and 41 minutes.

Frequency = frequency density × class width.
20 to 30: 1 × 10 = 10
30 to 40: 5 × 10 = 50
40 to 41: 5 × 1 = 5
Total = 65
Answer: 65

6 Series in context (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A new car costs £12000. Its value falls by 10% each year.
(a) Find its value after 6 years (nearest £).
(b) In which year does it first fall below £3000?

(a) Value = $12000 \times 0.9^6 = £6{,}377$
(b) $12000 \times 0.9^n < 3000$, so $n \lg 0.9 < \lg(3000 \div 12000)$
$n > \lg(3000 \div 12000) \div \lg 0.9 = 13.158$ (dividing by a negative number)
So n = 14
Answer: (a) £6,377 (b) 14 years

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7 Rates of change (connected rates) (All boards)

The radius of a circular oil slick increases at 0.3 cm per second.
Find the rate at which its area is increasing when r = 10 cm, in terms of π.

$A = \pi r^2$, so $\frac{dA}{dr} = 2\pi r$

$\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt} = 2\pi r \times 0.3$

When r = 10: $\frac{dA}{dt} = 6\pi$

Answer: 6π cm^2/s

8 Quadratics in a^x (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve the equation $5^{2x} - 4\times5^x - 5 = 0$.

Let $y = 5^x$. Then $5^{2x} = y^2$, so $y^2 - 4y - 5 = 0$
$(y - 5)(y + 1) = 0$
$y = 5$ or $y = -1$
$5^x > 0$ for all x, so $y = -1$ is rejected. $5^x = 5$ gives $x = 1$.
Answer: x = 1

9 Projectile at an angle: time and range (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A javelin is projected from level ground with horizontal velocity component 30 m s^{-1} and vertical velocity component 14.7 m s^{-1} upwards.
Take g = 9.8 m s^{-2}.
Find (a) the time of flight, (b) the horizontal range.

(a) Vertical: $0 = 14.7\,t - 4.9t^2$, so $t = 14.7 \div 4.9$
(b) Range = horizontal speed × time = 30 × 3
Answer: (a) 3 s (b) 90 m

10 Implicit differentiation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has equation $x^2 + xy + 3y^2 + 2y - x = 33$.
Find $\frac{dy}{dx}$ in terms of x and y, and the gradient at (−2, 3).

Differentiate with respect to x: $x\frac{dy}{dx} + 2x + 6y\frac{dy}{dx} + y + 2\frac{dy}{dx} - 1 = 0$

$\frac{dy}{dx} = \frac{1 - 2x - y}{x + 6y + 2}$

At (−2, 3): gradient = $\frac{1}{9}$

Answer: $\frac{dy}{dx} = \frac{1 - 2x - y}{x + 6y + 2}$; gradient $\frac{1}{9}$