Day 197 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Mean and standard deviation from sums (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A summary of the times, t minutes, taken by 9 people to complete a puzzle gives $\Sigma t = 260$ and $\Sigma t^2 = 8582$.
Find the mean and the standard deviation (3 s.f.).

Mean = $260 \div 9 = 28.9$

Variance = $\frac{8582}{9} - (28.889)^2 = 119.0$

Standard deviation = $\sqrt{119.0} = 10.9$

Answer: mean 28.9, standard deviation 10.9

2 Simplifying surds (AQA, Edexcel, OCR, MEI, Eduqas)

Simplify $\sqrt{294} - \sqrt{24} - \sqrt{96}$, giving your answer in the form $k\sqrt{6}$.

$\sqrt{294} = 7\sqrt{6}$, $\sqrt{24} = 2\sqrt{6}$, $\sqrt{96} = 4\sqrt{6}$

$7\sqrt{6} - 2\sqrt{6} - 4\sqrt{6} = \sqrt{6}$

Answer: $\sqrt{6}$

3 Derivatives of tan, sec, cosec and cot (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Differentiate $y = 5 \operatorname{cosec} 3x$ with respect to x,

and find the exact gradient when $x = \frac{\pi}{12}$.

Use the standard results for tan, sec, cosec and cot with the chain rule

$\frac{dy}{dx} = -15 \cot 3x \operatorname{cosec} 3x$

At $x = \frac{\pi}{12}$: $\frac{dy}{dx} = -15\sqrt{2}$

Answer: $\frac{dy}{dx} = -15 \cot 3x \operatorname{cosec} 3x$; gradient $-15\sqrt{2}$

4 Lift: weighing scales reading (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A woman of mass 75 kg stands on weighing scales in a lift. The lift decelerates at 0.5 m s^{-2} while moving upwards.
Take g = 9.8 m s^{-2}.
Find the force exerted on the woman by the scales.

The acceleration is downwards, magnitude 0.5

Taking up as positive: $R - 75 \times 9.8 = -75 \times 0.5$

$R = 75(9.8 - 0.5)$

Answer: 697.5 N

5 Integration by substitution (given substitution) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Use the substitution $u = \ln x$ to find the exact value of

$\int_1^{e^2} \frac{(\ln x)^3}{x} dx$.

$\frac{du}{dx} = \frac{1}{x}$, so $\frac{1}{x} dx = du$

Limits: $x = 1$ gives $u = 0$, $x = e^2$ gives $u = 2$

$\int_0^2 u^3 du = [\frac{u^4}{4}]_0^2$

$= 4$

Answer: 4

6 Finding the equation of a cubic from its graph (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows a sketch of the curve y = f(x), where f is a cubic. The curve meets the axes at the labelled points. Find f(x) in factorised form.

The curve touches the x-axis at x = 4 (repeated root) and crosses at x = −1

So $y = k(x - 4)^2(x + 1)$

The curve passes through (0, 32): $k \times 16 \times 1 = 32$

So k = 2

Answer: $f(x) = 2(x - 4)^2(x + 1)$

7 Rough table and a hanging particle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A block of mass 9 kg lies on a rough horizontal table (coefficient of friction 0.4). It is joined by a light string over a smooth pulley to a particle of mass 9 kg hanging freely.

Take g = 9.8 m s^{-2}.

The system is released. Find the acceleration and the tension in the string.

Block: $T - 0.4 \times 9\,g = 9\,a$

Hanging: $9\,g - T = 9\,a$

Add: $9\,g - 3.6\,g = 18\,a$

a = 2.94

$T = 9(9.8 - 2.94)$

Answer: $a = 2.94$ m s^{-2}; $T = 61.74$ N

8 Reciprocal graph: asymptotes and intercepts (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has equation $y = \frac{-2}{x + 1} - 1$.

(a) Write down the asymptotes.

(b) Find where the curve crosses the axes.

(a) The denominator is zero when x = −1, and $y \to -1$ as $x \to \pm\infty$

Asymptotes: x = −1 and y = −1

(b) x = 0: y = −2 − 1 = −3

y = 0: $\frac{-2}{x + 1} = 1$, so x + 1 = −2, x = −3

Answer: (a) x = −1, y = −1 (b) (0, −3) and (−3, 0)

9 Hypothesis test for correlation (AQA, Edexcel, OCR, MEI, Eduqas)

8 pairs of age of a house and its energy cost give r = 0.660.

One-tailed critical value (5%): 0.6215.

Test for positive correlation.

H_0: ρ = 0, H_1: ρ > 0

r = 0.660, critical value 0.6215; r is beyond the critical value in the right direction

r is in the critical region, so reject H_0

Conclusion: there is evidence of positive correlation between age of a house and its energy cost

Answer: reject H_0: there is evidence of positive correlation between age of a house and its energy cost

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10 Position vectors and dividing a line in a ratio (All boards)

A has position vector $\begin{pmatrix} -2 \\ 4 \end{pmatrix}$ and B has position vector $\begin{pmatrix} -2 \\ -2 \end{pmatrix}$.

The point P lies on AB with AP : PB = 1 : 2.

Find the position vector of P.

$\text{AB} = \begin{pmatrix} 0 \\ -6 \end{pmatrix}$

$\text{AP} = \frac{1}{3}\text{AB} = \begin{pmatrix} 0 \\ -2 \end{pmatrix}$

$\text{OP} = \text{OA} + \text{AP} = \begin{pmatrix} -2 \\ 2 \end{pmatrix}$

Answer: $\begin{pmatrix} \mathbf{-2} \\ \mathbf{2} \end{pmatrix}$