Day 196 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Constant acceleration (suvat) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A train moves in a straight line with constant acceleration. Its final speed is 19 m s^{-1}, its acceleration is 3 m s^{-2} and the motion lasts 4 s.
Find the initial speed.

$v = u + at$
$19 = u + 3 \times 4$
$u = 19 - 12$
Answer: 7 m s^{-1}

2 Standard integrals (definite) (All boards)

Find the exact value of

$\int_0^1 (3x + 4e^{2x})\,dx$.

$\int (3x + 4e^{2x})\,dx = \frac{3x^2}{2} + 2e^{2x}$

$= [\frac{3x^2}{2} + 2e^{2x}]_0^1$

$= 2e^2 - \frac{1}{2}$

Answer: $2e^2 - \frac{1}{2}$

3 Sigma notation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Evaluate $\sum_{r=7}^{20}(6r + 3)$.

There are $20 - 7 + 1 = 14$ terms
First term = 45, last term = 123
$S = \frac{14}{2}(45 + 123) = 1176$
Answer: 1176

4 Pulling at an angle: smooth surface (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A sledge of mass 5 kg is pulled across smooth horizontal ice by a rope with tension 30 N, inclined at α above the horizontal, where $\tan\alpha = \frac{4}{3}$.

Take g = 9.8 m s^{-2}.
Find (a) the acceleration of the sledge, (b) the normal reaction of the ice.

$\cos\alpha = 3/5$, $\sin\alpha = 4/5$
(a) Horizontal: $30 \times 3/5 = 5a$
(b) Vertical: $R + 30 \times 4/5 = 5 \times 9.8$
Answer: (a) 3.6 m s^{-2} (b) 25 N

5 Factor theorem with an unknown (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = 2x^3 + kx + 12$.
Given that $(x + 3)$ is a factor of f(x),
(a) find the value of k,
(b) factorise f(x) completely.

$f(-3) = 0$: $-54 - 3k + 12 = 0$
$k = -14$
$f(x) = (x + 3)(2x^2 - 6x + 4)$
$= 2(x - 2)(x - 1)(x + 3)$
Answer: (a) $k = -14$ (b) $f(x) = 2(x - 2)(x - 1)(x + 3)$

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6 Venn diagram with independent events (All boards)

The Venn diagram shows probabilities for events A and B, which are independent.
Find the values of x and y.

P(A) = 0.175 + 0.075 = 0.25 and P(A ∩ B) = 0.075
Independent: P(A ∩ B) = P(A)P(B), so P(B) = 0.075 ÷ 0.25 = 0.3
x = P(B) − P(A ∩ B) = 0.3 − 0.075 = 0.225
y = 1 − (0.175 + 0.075 + 0.225) = 0.525
Answer: x = 0.225, y = 0.525

7 Solving index equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $4^{2x+4} = 2^{x-1}$.

Write both sides as powers of 2: $2^{2(2x+4)} = 2^{x-1}$
Equate the indices: 4x + 8 = x − 1
3x = −9
x = −3
Answer: x = −3

8 Binomial: smallest n (All boards)

Given that each attempt at a task succeeds independently with probability 0.2, find the least number of attempts needed for the probability of at least one success to exceed 0.99.

P(at least one) = $1 - 0.8^n > 0.99$
$0.8^n < 0.01$
$n > \frac{\log 0.01}{\log 0.8} = 20.64$
Answer: n = 21

9 R cos(θ − α) form: solving (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Given that 5 cos θ + 12 sin θ ≡ 13 cos(θ − 67.4°),
solve 5 cos θ + 12 sin θ = 4 for 0 ≤ θ ≤ 360°. (1 d.p.)

13 cos(θ − 67.4°) = 4, so $\cos(\theta - 67.4°) = \frac{4}{13}$
θ − 67.4° = ±72.1°
θ = 67.4° + 72.1° = 139.5°
θ = 67.4° − 72.1° = −4.7°, so θ = 355.3°
Answer: θ = 139.5°, 355.3°

10 Derivatives of tan, sec, cosec and cot (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Differentiate y = 3 cot x with respect to x,
and find the exact gradient when $x = \frac{\pi}{6}$.

Use the standard results for tan, sec, cosec and cot with the chain rule
$\frac{dy}{dx} = -3\,\text{cosec}^2 x$
At $x = \frac{\pi}{6}$: $\frac{dy}{dx} = -12$
Answer: $\frac{dy}{dx} = -3\,\text{cosec}^2 x$; gradient −12