Day 195 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Driving force and resistance (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A van of mass 1500 kg moves along a straight horizontal road. The engine produces a driving force of 1800 N and the resistance to motion is 300 N.
Find the acceleration of the van.

Resultant force = 1800 − 300 = 1500 N
F = ma: 1500 = 1500 a
Answer: 1 m s^{-2}

2 Chain rule (All boards)

Given that $y = (2x - 4)^6$,
find $\frac{dy}{dx}$, and the gradient of the curve when x = 3.

Chain rule: differentiate the outer function, then multiply by the derivative of the inner function
$\frac{dy}{dx} = 384(x - 2)^5$
When x = 3: gradient = 384
Answer: $\frac{dy}{dx} = 384(x - 2)^5$; gradient 384

3 Describing transformations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Describe the single transformation that maps the graph of y = f(x) onto the graph of:
(a) y = f(−x)
(b) y = f(x) − 1
(c) y = 4f(x)

(a) y = f(−x) is a reflection in the y-axis.
(b) y = f(x) − 1 is a translation $\begin{pmatrix} 0 \\ -1 \end{pmatrix}$.
(c) y = 4f(x) is a stretch, scale factor 4, parallel to the y-axis.
Answer: (a) reflection in the y-axis (b) translation $\begin{pmatrix} 0 \\ -1 \end{pmatrix}$ (c) stretch, scale factor 4, parallel to the y-axis

4 Poisson distribution (CIE)

X ~ Po(0.5), modelling the number of calls to a helpline per minute.
Find, to 3 significant figures,
(a) P(X ≤ 2)
(b) P(X = 2)

(a) P(X ≤ 2) = P(X = 0) + ... + P(X = 2) = 0.986
(b) $P(X = 2) = e^{-0.5} \times \frac{0.5^2}{2!} = 0.0758$
Answer: (a) 0.986 (b) 0.0758

5 Inequalities with exponents and logs (All boards)

£1000 is invested at 4% compound interest per year.
Find the least whole number of years for the investment to exceed £5000.

$1000 \times 1.04^n > 5000$, so $1.04^n > 5$
$n > \frac{\ln 5}{\ln 1.04} = 41.04$
The least whole number is n = 42
Answer: 42 years

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6 Venn diagram with independent events (All boards)

The Venn diagram shows probabilities for events A and B, which are independent.
Find the values of x and y.

P(A) = 0.45 + 0.15 = 0.6 and P(A ∩ B) = 0.15
Independent: P(A ∩ B) = P(A)P(B), so P(B) = 0.15 ÷ 0.6 = 0.25
x = P(B) − P(A ∩ B) = 0.25 − 0.15 = 0.1
y = 1 − (0.45 + 0.15 + 0.1) = 0.3
Answer: x = 0.1, y = 0.3

7 R cos(θ − α) form: solving (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Given that $2\cos\theta + 3\sin\theta \equiv \sqrt{13}\cos(\theta - 56.3°)$,
solve $2\cos\theta + 3\sin\theta = 3$ for $0 \le \theta \le 360°$. (1 d.p.)

$\sqrt{13}\cos(\theta - 56.3°) = 3$, so $\cos(\theta - 56.3°) = \frac{3}{\sqrt{13}}$

$\theta - 56.3° = \pm 33.7°$
$\theta = 56.3° + 33.7° = 90.0°$
$\theta = 56.3° - 33.7° = 22.6°$
Answer: θ = 22.6°, 90.0°

8 Beam about to tilt (AQA, Edexcel, OCR, MEI, Eduqas)

A uniform plank AB of length 8 m and mass 20 kg rests horizontally on supports at C and D, where AC = 0.5 m and DB = 2 m.
A girl of mass 20 kg walks from D towards B. Find how far she is from D when the plank is about to tilt.

About to tilt about D: reaction at C is zero
Moments about D: 20 g × (6 − 4) = 20 g × x
20 × 2 = 20 x
Answer: 2 m

9 Rationalising the denominator (AQA, Edexcel, OCR, MEI, Eduqas)

Express $\frac{12}{2-\sqrt{5}}$ in the form $p + q\sqrt{5}$, where p and q are integers.

Multiply top and bottom by the conjugate $(2 + \sqrt{5})$:
Denominator: $(2 - \sqrt{5})(2 + \sqrt{5}) = 2^2 - 1^2 \times 5 = -1$
Numerator: $(12)(2 + \sqrt{5}) = 24 + 12\sqrt{5}$
So the fraction = $\frac{24 + 12\sqrt{5}}{-1} = -24 - 12\sqrt{5}$
Answer: $-24 - 12\sqrt{5}$

10 Parametric to Cartesian (AQA, Edexcel, OCR, MEI, Eduqas)

A curve has parametric equations $x = 4t^2$, $y = 3t$, $t \in \mathbb{R}$.
Find a Cartesian equation of the curve.

From $y = 3t$: $t = \frac{y}{3}$
$x = 4(\frac{y}{3})^2 = \frac{4y^2}{9}$
So $y^2 = \frac{9}{4}x$
Answer: $y^2 = \frac{9}{4}x$