**Day 194** **A Level Maths: B Tier 10-a-day: Model Solutions**

## 1 Speed and direction from a velocity vector (AQA, Edexcel, OCR, MEI, Eduqas)

A remote-control car moves with constant velocity $(-40\mathbf{i} + 42\mathbf{j})$ m s$^{-1}$, where $\mathbf{i}$ and $\mathbf{j}$ are unit vectors east and north.
Find its speed and the acute angle its direction makes with $\mathbf{i}$, to 1 d.p.

Speed = $\sqrt{(-40^2 + 42^2)} = 58$
$\tan\theta = 42 \div 40$
**Answer: 58 m s$^{-1}$; 46.4°**

## 2 3D vectors: magnitude, distance and unit vectors (All boards)

The vector $\mathbf{a} = \begin{pmatrix} -12 \\ 1 \\ 12 \end{pmatrix}$.
Find $|\mathbf{a}|$ and a unit vector in the direction of $\mathbf{a}$.

$|\mathbf{a}| = \sqrt{12^2 + 1^2 + 12^2} = \sqrt{289} = 17$
Unit vector = $\frac{1}{17}\mathbf{a}$
**Answer: $|\mathbf{a}| = 17$, unit vector $\frac{1}{17}\begin{pmatrix} -12 \\ 1 \\ 12 \end{pmatrix}$**

## 3 Parametric to Cartesian (AQA, Edexcel, OCR, MEI, Eduqas)

A curve has parametric equations $x = 4t^2$, $y = 5t$, $t \in \mathbb{R}$.
Find a Cartesian equation of the curve.

From $y = 5t$: $t = \frac{y}{5}$
$x = 4(\frac{y}{5})^2 = \frac{4y^2}{25}$
So $y^2 = \frac{25}{4}x$
**Answer: $y^2 = \frac{25}{4}x$**

## 4 Exact values of sec, cosec and cot (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Given that $\cot\theta = \frac{3}{4}$ and $\theta$ is acute,
find the exact value of $\sec\theta$.

Use $1 + \tan^2\theta = \sec^2\theta$ and $1 + \cot^2\theta = \operatorname{cosec}^2\theta$
$\tan\theta = \frac{4}{3}$, so $\sec^2\theta = 1 + \frac{16}{9} = \frac{25}{9}$
$\sec\theta = \frac{5}{3}$ (positive as $\theta$ is acute)
**Answer: $\frac{5}{3}$**

## 5 Tree diagram (All boards)

The tree diagram shows probabilities for event E, followed by event F.
Find the probability that exactly one of E and F occurs.

P(E and not F) = 0.6 × 0.7 = 0.42
P(not E and F) = 0.4 × 0.9 = 0.36
Exactly one: 0.42 + 0.36 = 0.78
**Answer: 0.78**

**Day 194** **A Level Maths: B Tier 10-a-day: Model Solutions**

## 6 Exponential quadratics and ln equations (All boards)

Solve $e^{2x} - 15e^x + 36 = 0$,
giving your answers as exact values.

Let $y = e^x$: $y^2 - 15y + 36 = 0$
$(y - 3)(y - 12) = 0$, so $e^x = 3$ or $e^x = 12$
$x = \ln 3$ or $x = \ln 12$
**Answer: $x = \ln 3$, $x = \ln 12$**

## 7 Differentiating position vectors (AQA, Edexcel, OCR, MEI, Eduqas)

A particle moves so that its position vector relative to O at time t s is $\mathbf{r} = (3t^2)\mathbf{i} + (2t^3 + 2t)\mathbf{j}$ metres.
Find the velocity when t = 1, and the speed at that time.

$\mathbf{v} = d\mathbf{r}/dt = (6t)\mathbf{i} + (6t^2 + 2)\mathbf{j}$
$t = 1$: $\mathbf{v} = (6)\mathbf{i} + (8)\mathbf{j}$
Speed $= \sqrt{(6^2 + 8^2)}$
**Answer: $\mathbf{v} = (6)\mathbf{i} + (8)\mathbf{j}$ m s$^{-1}$; speed 10 m s$^{-1}$**

## 8 Binomial one-tailed test (All boards)

A manufacturer claims that a proportion 0.3 of its sweets are red. A sample of 25 sweets has 13 red.
Test, at the 5% level, whether the true proportion is higher than claimed.

$H_0$: $p = 0.3$, $H_1$: $p > 0.3$. Let $X \sim B(25, 0.3)$ under $H_0$
$P(X \geq 13) = 1 - P(X \leq 12) = 0.0175$
$0.0175 < 0.05$, so reject $H_0$
There is sufficient evidence that the proportion is higher than claimed
**Answer: reject $H_0$: evidence that the proportion is higher**

## 9 Geometric series and logarithms (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A geometric series has first term 5 and common ratio 4.
Find the smallest value of n for which the sum of the first n terms exceeds 50000.

$S_n = \frac{5(4^n - 1)}{3} > 50000$
$4^n > 30001$
$n > \frac{\lg 30001}{\lg 4} = 7.436$
So the smallest integer n is 8
**Answer: $n = 8$**

## 10 Equation of a circle from given information (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A circle has centre $(-4, 4)$ and passes through the point $(0, 1)$.
Find an equation of the circle.

$r^2 = (0 - (-4))^2 + (1 - 4)^2$
$= 16 + 9 = 25$
**Answer: $(x + 4)^2 + (y - 4)^2 = 25$**