Day 193 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Interpreting regression (AQA, Edexcel, OCR, MEI, Eduqas)

Regression line (x from 1 to 9): $y = 12 + 25x$.
x is the hours of sunshine, y the number of ice creams sold.
(a) Interpret the gradient.
(b) Estimate y when x = 2 and comment.

(a) The gradient: for each extra hour of sunshine, y is predicted to increase by 25
(b) $y = 12 + (25) \times 2 = 62$
x = 2 lies within the range of the data (interpolation), so the estimate is reliable
Answer: (a) y increases by 25 per extra hour of sunshine (b) 62, reliable (interpolation)

2 Completing the square (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Write $x^2 - 10x + 34$ in the form $a(x + p)^2 + q$.
(b) Write down the coordinates of the minimum point of $y = x^2 - 10x + 34$.

$x^2 - 10x = (x - 5)^2 - 25$
So $x^2 - 10x + 34 = (x - 5)^2 + 9$
$(x - 5)^2 \geq 0$ with equality at x = 5, so the minimum is 9 at x = 5.
Answer: (a) $(x - 5)^2 + 9$ (b) (5, 9)

3 Derivatives of tan, sec, cosec and cot (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Differentiate $y = \tan x$ with respect to x,
and find the exact gradient when $x = \frac{\pi}{4}$.

Use the standard results for tan, sec, cosec and cot with the chain rule
$\frac{dy}{dx} = \sec^2 x$
At $x = \frac{\pi}{4}$: $\frac{dy}{dx} = 2$
Answer: $\frac{dy}{dx} = \sec^2 x$; gradient 2

4 Right-angled triangle on a grid (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The points A, B and C are (3, −7), (0, −3) and (−8, −9).
(a) Show that angle ABC = 90°.
(b) Find the area of triangle ABC.

(a) Gradient AB $= \frac{-4}{3} = -\frac{4}{3}$
Gradient BC $= \frac{-6}{-8} = \frac{3}{4}$
$(-\frac{4}{3}) \times \frac{3}{4} = -1$, so AB and BC are perpendicular.
(b) AB = 5 and BC = 10
Area $= \frac{1}{2} \times AB \times BC = 25$
Answer: (b) area = 25

5 Normal curve and shaded area (All boards)

$X \sim N(60, 4^2)$. The shaded area on the diagram represents the probability that X lies between two values.
Find this probability to 3 significant figures.

Shaded area = P(62.00 < X < 66.00)
= P(X < 66.00) − P(X < 62.00)
= 0.9332 − 0.6915 = 0.242
Answer: 0.242

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6 Separable differential equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $\frac{dy}{dx} = (5 + y)e^{-x}$,

given that $y = 3$ when $x = 0$.

$\frac{1}{5+y}\,dy = e^{-x}\,dx$, so $\ln(5 + y) = -e^{-x} + c$

$x = 0$: $\ln 8 = -1 + c$, so $c = 1 + \ln 8$

$5 + y = 8\,e^{1 - e^{-x}}$, so $y = 8\,e^{1 - e^{-x}} - 5$

Answer: $y = 8e^{1 - e^{-x}} - 5$

7 Pulley system: speed on reaching the ground (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Particles A (2 kg) and B (8 kg) are joined by a light string over a smooth pulley. Both are held with the string taut, B is 6 m above the ground, then released.

Take $g = 9.8$ m s^{-2}.

Find the speed with which B hits the ground.

$a = (8 - 2)g \div (8 + 2) = 5.88$

$v^2 = 0 + 2 \times 5.88 \times 6 = 70.56$

$v = 8.4$

Answer: 8.4 m s^{-1}

8 Rates of change (connected rates) (All boards)

The radius of a circular oil slick increases at 0.5 cm per second.

Find the rate at which its area is increasing when $r = 5$ cm, in terms of π.

$A = \pi r^2$, so $\frac{dA}{dr} = 2\pi r$

$\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt} = 2\pi r \times 0.5$

When $r = 5$: $\frac{dA}{dt} = 5\pi$

Answer: 5π cm^2/s

9 Vertical projection upwards (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A marble is projected vertically upwards with speed 26 m s^{-1} from ground level.

Take $g = 9.8$ m s^{-2}. Find, to 3 significant figures,

(a) the greatest height reached, (b) the time of flight.

(a) $0 = 26^2 - 2 \times 9.8 \times H$, so $H = 676 \div 19.6$

(b) $s = 0$: $t = 2 \times 26 \div 9.8$

Answer: (a) 34.5 m (b) 5.31 s

10 Using a binomial expansion to estimate (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Expand $(1 + 3x)^{-2}$ up to the x^2 term.

(b) Use $x = 0.01$ to estimate $\frac{1}{1.03^2}$ to 5 decimal places.

(a) $(1 + ax)^n = 1 + n(ax) + \frac{n(n-1)}{2}(ax)^2 + \ldots$ with $n = -2$, $a = 3$:

$= 1 - 6x + 27x^2$

(b) $1 + 3 \times 0.01 = 1.03$, so substitute $x = 0.01$:

$1 - 6 \times 0.01 + 27 \times 0.01^2 = 0.94270$

Answer: (a) $1 - 6x + 27x^2 + \ldots$ (b) 0.94270