Day 192 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Linear inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $6x - 9 < -4x$.

$10x < 9$

Divide by 10: $x < \frac{9}{10}$

Answer: $x < \frac{9}{10}$

2 Arc length and sector area (radians) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A sector of a circle has area 135.2 cm^2 and angle 1.6 radians.
Find (a) the radius, (b) the perimeter of the sector.

$\frac{1}{2}r^2\theta = 135.2$, so $r^2 = \frac{2 \times 135.2}{1.6} = 169$, $r = 13$

Perimeter $= 2r + r\theta = 26 + 20.8 = 46.8$ cm

Answer: (a) 13 cm (b) 46.8 cm

3 Smooth inclined plane (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A block slides from rest down a line of greatest slope of a smooth plane inclined at α, where $\sin\alpha = \frac{3}{5}$ to the horizontal.

Take $g = 9.8$ m s^{-2}.
Find its acceleration, and the distance it travels in 3 s.

Along the plane: $mg\sin\alpha = ma$, so $a = 9.8 \times 0.6 = 5.88$

$s = \frac{1}{2}at^2 = \frac{1}{2} \times 5.88 \times 3^2$

Answer: 5.88 m s^{-2}; 26.46 m

4 Right-angled triangle on a grid (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The points A, B and C are (−4, −5), (−3, −2) and (0, −3).
(a) Show that angle ABC = 90°.
(b) Find the area of triangle ABC.

(a) Gradient AB $= \frac{-3}{-1} = 3$

Gradient BC $= \frac{-1}{3} = -\frac{1}{3}$

$3 \times (-\frac{1}{3}) = -1$, so AB and BC are perpendicular.

(b) AB $= \sqrt{10}$ and BC $= \sqrt{10}$

Area $= \frac{1}{2} \times AB \times BC = 5$

Answer: (b) area = 5

5 Scatter diagram (AQA, Edexcel, OCR, MEI, Eduqas)

The scatter diagram shows eight pairs of values of x and y.
(a) Describe the correlation.
(b) Why should $y = 5 + x$ not be used for $x = 14$?

(a) The points lie loosely around a straight line rising: weak positive correlation
(b) The data cover $x = 1$ to 10; $x = 14$ is outside this range
so using the line would be extrapolation and may be unreliable
Answer: (a) weak positive correlation (b) extrapolation outside the range of the data

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6 Perpendicular bisector (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A is the point (8, 2) and B is the point (7, −4).
Find the perpendicular bisector of AB as ax + by + c = 0, with integer a, b, c.

Midpoint of AB = $(\frac{15}{2}, -1)$

Gradient of AB = $\frac{-6}{-1} = 6$

Perpendicular gradient = $-\frac{1}{6}$

$y + 1 = -\frac{1}{6}(x - \frac{15}{2})$, which gives $2x + 12y - 3 = 0$

Answer: $2x + 12y - 3 = 0$

7 Small angle approximations (AQA, Edexcel, OCR, MEI, Eduqas)

For small θ (in radians), use $\sin\theta \approx \theta$ and $\cos\theta \approx 1 - \frac{\theta^2}{2}$

to find the value of $\lim_{\theta \to 0} \frac{1 - \cos 4\theta}{\theta \sin 3\theta}$.

$1 - \cos 4\theta \approx 1 - (1 - \frac{4^2\theta^2}{2}) = \frac{16\theta^2}{2}$

$\theta \sin 3\theta \approx \theta \times 3\theta = 3\theta^2$

Ratio = $\frac{16\theta^2}{(2 \times 3\theta^2)} = \frac{8}{3}$

Answer: $\frac{8}{3}$

8 Maximum speed from acceleration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves in a straight line. At time t s its velocity is $v = 48t - 6t^2$ m s^{-1}, for $t \geq 0$.
Find the maximum speed of the particle and the time at which it occurs.

$a = dv/dt = 48 - 12t$
Maximum speed when a = 0: $t = 48 \div 12 = 4$
$v = 48 \times 4 - 6 \times 4^2$
Answer: 96 m s^{-1} at t = 4 s

9 Normal approximation to the binomial (AQA, OCR, CIE)

X ~ B(60, 0.5). Use a normal approximation to find $P(29 \leq X \leq 35)$, to 3 significant figures.

$Y \sim N(np, np(1-p)) = N(30.00, 15.00)$, so $\sigma = \sqrt{15.00} = 3.873$
Use a continuity correction.
$P(29 \leq X \leq 35) \approx P(28.5 < Y < 35.5) = 0.573$
Answer: 0.573

10 Newton-Raphson method (AQA, Edexcel, OCR, MEI, Eduqas)

f(x) = ln x + x − 3. Use the Newton-Raphson method with
$x_0 = 2$ to find x_1 and x_2, to 4 d.p.

$f'(x) = 1 + \frac{1}{x}$

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$x_1 = 2 - \frac{-0.3069}{1.500} = 2.2046$

$x_2 = 2.2046 - \frac{-0.004900}{1.454} = 2.2079$

Answer: $x_1 = 2.2046$, $x_2 = 2.2079$