Day 191 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Arc length and sector area (radians) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows a sector OAB of a circle with centre O, radius 8 cm and angle 1.5 radians.
Find (a) the arc length AB, (b) the area of the sector.

Arc length = $r\theta = 8 \times 1.5 = 12$ cm

Area = $\frac{1}{2}r^2\theta = \frac{1}{2} \times 8^2 \times 1.5 = 48$ cm^2

Answer: (a) 12 cm (b) 48 cm^2

2 Series in context (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A charity raises £250 in week 1. In each following week it raises £15 more than in the previous week.
(a) Find the amount raised in week 10.
(b) Find the total for the first 24 weeks.

(a) Arithmetic sequence with a = 250, d = 15
$u_{10} = 250 + 9 \times 15 = £385$

(b) $S_{24} = \frac{24}{2}(2 \times 250 + 23 \times 15) = £10{,}140$

Answer: (a) £385 (b) £10,140

3 Product moment correlation coefficient (AQA, Edexcel, OCR, MEI, Eduqas)

For 11 pairs of data, $S_{xx} = 80$, $S_{yy} = 125$ and $S_{xy} = -47$.
Find the PMCC (3 s.f.) and describe the correlation.

$r = \frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}} = \frac{-47}{\sqrt{80 \times 125}}$

$= \frac{-47}{100.00} = -0.470$

r is negative, so there is weak negative correlation

Answer: r = −0.470, weak negative correlation

4 Arithmetic sequences and series (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

An arithmetic sequence has 4th term 38 and 11th term 80.
(a) Find the first term and the common difference.
(b) Find the sum of the first 25 terms.

a + 3d = 38 and a + 10d = 80
Subtract: 7d = 42, so d = 6 and a = 20

$S_{25} = \frac{25}{2}(2 \times 20 + 24 \times 6) = 2300$

Answer: (a) a = 20, d = 6 (b) 2300

5 Inverse trigonometric functions (AQA, Edexcel, OCR, MEI, Eduqas)

Find the exact value, in radians, of

(a) $\arccos(-\frac{1}{2})$ (b) $\arctan(\sqrt{3})$

(a) arccos lies in $[0, \pi]$, and $\cos(\frac{2\pi}{3}) = -\frac{1}{2}$, so the value is $\frac{2\pi}{3}$

(b) arctan lies in $(-\frac{\pi}{2}, \frac{\pi}{2})$, and $\tan(\frac{\pi}{3}) = \sqrt{3}$, so the value is $\frac{\pi}{3}$

Answer: (a) $\frac{2\pi}{3}$ (b) $\frac{\pi}{3}$

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6 Binomial two-tailed test (All boards)

A factory says a proportion 0.3 of its parts are defective. In a sample of 20 parts, 11 are defective. Test, at the 5% significance level, whether the proportion is different from 0.3.

H_0: $p = 0.3$, H_1: $p \neq 0.3$. Under H_0, $X \sim B(20, 0.3)$

$P(X \geq 11) = 1 - P(X \leq 10) = 0.0171$

Two-tailed: compare with 0.025. $0.0171 < 0.025$, so reject H_0

There is sufficient evidence that the proportion is different from 0.3

Answer: reject H_0: sufficient evidence that the proportion is different from 0.3

7 Projection from a height (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A ball is thrown vertically upwards at 9.8 m s^{-1} from a point 14.7 m above horizontal ground. Take $g = 9.8$ m s^{-2}. Find the time before it hits the ground.

Taking up as positive: $-14.7 = 9.8\,t - 4.9t^2$

$4.9t^2 - 9.8\,t - 14.7 = 0$

$t = (9.8 \pm \sqrt{384.16}) \div 9.8 = (9.8 \pm 19.6) \div 9.8$

Positive root only.

Answer: 3 s

8 Collinear points and parallel vectors (All boards)

Points A, B and C have position vectors $\begin{pmatrix} -2 \\ -3 \\ 1 \end{pmatrix}$, $\begin{pmatrix} -2 \\ -6 \\ -2 \end{pmatrix}$ and $\begin{pmatrix} -2 \\ -12 \\ r \end{pmatrix}$ respectively. Given that A, B, C are collinear, find r.

$\overrightarrow{AB} = \begin{pmatrix} 0 \\ -3 \\ -3 \end{pmatrix}$, $\overrightarrow{AC} = \begin{pmatrix} 0 \\ -9 \\ r - 1 \end{pmatrix}$

$\overrightarrow{AC} = \lambda \overrightarrow{AB}$: component 2 gives $-9 = \lambda \times -3$, so $\lambda = 3$

$r - 1 = 3 \times -3$, so $r = -8$

Answer: $r = -8$

9 Proof by contradiction (AQA, Edexcel, OCR, MEI, Eduqas)

Prove by contradiction that there are no integers m and n such that $8m + 6n = 105$.

Assume there are integers m and n with $8m + 6n = 105$.

$8m + 6n = 2(4m + 3n)$.

So the left-hand side is a multiple of 2.

But $105 = 2 \times 52 + 1$ is not a multiple of 2.

This is a contradiction, so no such integers exist.

Answer: no integer solutions (LHS is a multiple of 2, 105 is not)

10 Constant velocity: position vector (AQA, Edexcel, OCR, MEI, Eduqas)

A drone moves with constant velocity $(-\mathbf{i} - \mathbf{j})$ m s^{-1}. At time t = 0 its position vector relative to a fixed origin O is $(\mathbf{i} + 7\mathbf{j})$ m. Find (a) its position vector when t = 4, (b) the value of t when it crosses the line through O in the direction of $\mathbf{i}$.

(a) $\mathbf{r} = \mathbf{r}_0 + \mathbf{v}t = (\mathbf{i} + 7\mathbf{j}) + 4(-\mathbf{i} - \mathbf{j}) = (-3\mathbf{i} + 3\mathbf{j})$

(b) $\mathbf{j}$ component zero: $7 - t = 0$, so $t = 7$

Answer: (a) $(-3\mathbf{i} + 3\mathbf{j})$ m (b) t = 7 s