Day 190 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Pulling at an angle: smooth surface (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A sledge of mass 8 kg is pulled across smooth horizontal ice by a rope with tension 39 N, inclined at α above the horizontal, where $\tan\alpha = \frac{5}{12}$.

Take $g = 9.8$ m s^{-2}.

Find (a) the acceleration of the sledge, (b) the normal reaction of the ice.

$\cos\alpha = 12/13$, $\sin\alpha = 5/13$

(a) Horizontal: $39 \times 12/13 = 8a$

(b) Vertical: $R + 39 \times 5/13 = 8 \times 9.8$

Answer: (a) 4.5 m s^{-2} (b) 63.4 N

2 Stratified sampling (AQA, Edexcel, OCR, MEI, Eduqas)

A sports club has 20 junior members, 25 senior members and 80 veteran members.

A stratified sample of size 25 is taken. Find the number selected from each group.

Total = 20 + 25 + 80 = 125

Sampling fraction = $25/125 = \frac{1}{5}$

junior: $20 \div 5 = 4$

senior: $25 \div 5 = 5$

veteran: $80 \div 5 = 16$

Answer: junior 4, senior 5, veteran 16

3 Impulse of a constant force: v = u + at with forces (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A puck of mass 5 kg is moving on a smooth horizontal surface at 4 m s^{-1} when a constant horizontal force of 20 N, in the direction of motion, acts for 7 s.

Find the speed of the puck at the end of this time and the distance it moves.

$F = ma$: $a = 20 \div 5 = 4$

$v = u + at = 4 + 4 \times 7$

$s = ut + \frac{1}{2}at^2$

Answer: 32 m s^{-1}; 126 m

4 Proof by deduction (AQA, Edexcel, OCR, MEI, Eduqas)

Prove that $(2n + 2)^2 - (2n - 2)^2$ is a multiple of 16 for all integer values of n.

$(2n + 2)^2 = 4n^2 + 8n + 4$

$(2n - 2)^2 = 4n^2 - 8n + 4$

Difference $= 16n = 16 \times n$

n is an integer, so this is a multiple of 16 for every integer n.

Answer: difference = 16n, a multiple of 16

5 Position vectors and dividing a line in a ratio (All boards)

A has position vector $\begin{pmatrix}-4\\-4\end{pmatrix}$ and B has position vector $\begin{pmatrix}0\\-4\end{pmatrix}$.

The point P lies on AB with AP : PB = 3 : 1.

Find the position vector of P.

$\overrightarrow{AB} = \begin{pmatrix}4\\0\end{pmatrix}$

$\overrightarrow{AP} = \frac{3}{4}\overrightarrow{AB} = \begin{pmatrix}3\\0\end{pmatrix}$

$\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AP} = \begin{pmatrix}-1\\-4\end{pmatrix}$

Answer: $\begin{pmatrix}\mathbf{-1}\\\mathbf{-4}\end{pmatrix}$

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6 Coefficients in a binomial expansion (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the term independent of x in the expansion of

$(x^2 - \frac{3}{x})^6$.

The general term is $C(6, k)(x^2)^{6-k}(-3/x)^k$, with power of x equal to $12 - 3k$
Independent of x: $12 - 3k = 0$, so $k = 4$
Term = $15 \times (-3)^4 = 1215$
Answer: 1215

7 Fixed-point iteration (All boards)

(a) Show that $x^3 - 4x - 2 = 0$ can be written as $x = \sqrt[3]{4x + 2}$.
(b) Use $x_{n+1} = \sqrt[3]{4x_n + 2}$ with $x_0 = 2$ to find x_1, x_2, x_3 to 4 d.p.

(a) $x^3 = 4x + 2$, so $x = \sqrt[3]{4x + 2}$
(b) $x_1 = 2.1544$
$x_2 = 2.1979$
$x_3 = 2.2098$
Answer: $x_1 = 2.1544$, $x_2 = 2.1979$, $x_3 = 2.2098$

8 Binomial expansion (any power) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Expand $(4 - 3x)^{\frac{3}{2}}$ in ascending powers of x up to and including the term in x^3.
(b) State the range of values of x for which the expansion is valid.

Take out 4: $(4 - 3x)^{\frac{3}{2}} = 4^{\frac{3}{2}}(1 + -\frac{3}{4}x)^{\frac{3}{2}} = 8(1 + -\frac{3}{4}x)^{\frac{3}{2}}$

Expand $(1 + -\frac{3}{4}x)^{\frac{3}{2}} = 1 + n(ax) + \frac{n(n-1)}{2!}(ax)^2 + \frac{n(n-1)(n-2)}{3!}(ax)^3 + ...$

Multiplying by 8: coefficients of 1, x, x^2, x^3 are 8, -9, $\frac{27}{16}$, $\frac{27}{128}$

Valid when $|\frac{3}{4}x| < 1$, that is $|x| < \frac{4}{3}$

Answer: (a) $8 - 9x + \frac{27}{16}x^2 + \frac{27}{128}x^3 + ...$ (b) $|x| < \frac{4}{3}$

9 Derivatives of tan, sec, cosec and cot (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Differentiate $y = 3 \sec 4x$ with respect to x,

and find the exact gradient when $x = \frac{\pi}{12}$.

Use the standard results for tan, sec, cosec and cot with the chain rule
$\frac{dy}{dx} = 12 \tan 4x \sec 4x$

At $x = \frac{\pi}{12}$: $\frac{dy}{dx} = 24\sqrt{3}$

Answer: $\frac{dy}{dx} = 12 \tan 4x \sec 4x$; gradient $24\sqrt{3}$

10 Frequency table: mean and standard deviation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The table shows the number of siblings, x, of 54 students.
Calculate the mean and standard deviation (3 s.f.).

$\Sigma f = 54$, $\Sigma fx = 121$, $\Sigma fx^2 = 419$

Mean = $\frac{121}{54} = 2.24$

Variance = $\frac{419}{54} - (2.2407)^2 = 2.738$

Standard deviation = 1.65
Answer: mean 2.24, standard deviation 1.65