Day 19 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Combined inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the set of values of x for which
$5x - 1 \geq -7$ and $x^2 + 6x + 5 \geq 0$.

$5x - 1 \geq -7$ gives $x \geq -\frac{6}{5}$

$x^2 + 6x + 5 \geq 0$: $(x + 5)(x + 1) \geq 0$ gives $x \leq -5$ or $x \geq -1$
Both must hold: $x \geq -1$
Answer: $x \geq -1$

2 Power, driving force and resistance (CIE)

A car of mass 1200 kg moves along a horizontal road. The engine works at a constant rate of 48 kW. The resistance to motion is constant at 600 N.
Find the acceleration when the speed is 16 m s^{-1}.

$P = Fv$: $F = 48000 \div 16 = 3000$ N
$F - R = ma$: $3000 - 600 = 1200\,a$
Answer: 2 m s^{-2}

3 Inverse normal (All boards)

The length X cm of a metal rod is normal with mean 250, s.d. 10.
Find k (3 s.f.): (a) $P(X < k) = 0.2$,
(b) $P(X > k) = 0.01$.

(a) $z = -0.8416$, so $k = 250 + (-0.8416) \times 10 = 242$
(b) $P(X < k) = 0.99$, $z = 2.326$, so $k = 250 + 2.326 \times 10 = 273$
Answer: (a) 242 (b) 273

4 Triangle formed by two lines and an axis (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Line l_1 has equation $4x - 3y = 20$ and line l_2 has equation $4x - 5y = 28$.
The lines meet at P. Find the area of the triangle formed by l_1, l_2 and the x-axis.

Solve simultaneously: P = (2, −4)
l_1 meets $y = 0$ at $x = 5$, and l_2 meets $y = 0$ at $x = 7$
Base = 2, height = 4
Area = $\frac{1}{2} \times$ base $\times$ height = 4
Answer: 4

5 Tree diagram (All boards)

The tree diagram shows probabilities for event E, followed by event F.
Find the probability that exactly one of E and F occurs.

P(E and not F) = $0.2 \times 0.7 = 0.14$
P(not E and F) = $0.8 \times 0.5 = 0.4$
Exactly one: $0.14 + 0.4 = 0.54$
Answer: 0.54

6 Locating roots: change of sign (All boards)

The function $f(x) = 2\cos x - \frac{3}{2}x$.

Show that $f(x) = 0$ has a root between $x = 0.5$ and $x = 1$ (x in radians).

$f(0.5) = 1.01$
$f(1) = -0.419$
f is continuous and changes sign, so there is a root in the interval
Answer: $f(0.5) > 0$ and $f(1) < 0$: sign change

7 Particle on a table joined to a hanging particle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle of mass 3 kg lies on a smooth horizontal table. It is attached by a light string over a smooth pulley at the edge of the table to a particle of mass 2 kg hanging freely.

Take g = 9.8 m s^{-2}.

The system is released from rest. Find the acceleration and the tension in the string.

Hanging: $2g - T = 2a$

Table: $T = 3a$

$2g = 5a$, so $a = 3.92$

$T = 3 \times 3.92$

Answer: a = 3.92 m s^{-2}; T = 11.76 N

8 Periodic sequences (AQA, Edexcel, OCR, MEI, Eduqas)

A sequence is defined by $u_{n+1} = \frac{1}{1 - u_n}$, $u_1 = -2$.

(a) Show that the sequence is periodic, stating the period.

(b) Find u_{25} and $\sum_{r=1}^{25} u_r$.

(a) $u_2 = \frac{1}{3}$, $u_3 = \frac{3}{2}$, $u_4 = -2$

$u_4 = u_1$ so the sequence repeats with period 3

(b) $25 = 3 \times 8 + 1$, so $u_{25} = u_1 = -2$

Sum of one cycle = $-\frac{1}{6}$, so the sum = $8 \times (-\frac{1}{6}) - 2 = -\frac{10}{3}$

Answer: period 3; $u_{25} = -2$; sum = $-\frac{10}{3}$

9 Intersection of two lines in 3D (CIE)

Line l_1: $\mathbf{r} = \begin{pmatrix} -7 \\ -7 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}$.

Line l_2: $\mathbf{r} = \begin{pmatrix} -1 \\ -3 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$.

Show that l_1 and l_2 intersect and find the point.

i: $-7 + 2\lambda = -1 + 0\mu$

j: $-7 + 2\lambda = -3 + 1\mu$

Solving i and j: $\lambda = 3$, $\mu = 2$

k check: $4 + (-1)(3) = 1$ and $1 + 0(2) = 1$, equal

Point: $\begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix}$

Answer: $\begin{pmatrix} \mathbf{-1} \\ \mathbf{-1} \\ \mathbf{1} \end{pmatrix}$

10 Simplify using double angle formulae (All boards)

Simplify $\frac{\sin 4x}{\sin 2x}$,

giving your answer as a single trigonometric term.

$\sin 4x = 2 \sin 2x \cos 2x$

so the expression is $\frac{2 \sin 2x \cos 2x}{\sin 2x} = 2 \cos 2x$

Answer: 2 cos 2x