Day 189 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Chain rule (All boards)

Given that $y = \tan 5x$,

find $\frac{dy}{dx}$, and the gradient of the curve when $x = \frac{\pi}{20}$.

Chain rule: differentiate the outer function, then multiply by the derivative of the inner function

$\frac{dy}{dx} = 5\sec^2 5x$

When $x = \frac{\pi}{20}$: gradient = 10

Answer: $\frac{dy}{dx} = 5\sec^2 5x$; gradient 10

2 Arithmetic sequences and series (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the sum of all the multiples of 9 that lie between 21 and 336.

First multiple = 9 × 3 = 27, last multiple = 9 × 37 = 333

This is an arithmetic series with d = 9 and 35 terms

$S = \frac{35}{2}(27 + 333) = 6300$

Answer: 6300

3 Sector and arc in terms of π (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A sector of a circle has radius 13 cm and angle 270°.

Find the perimeter of the sector, in terms of π.

$\theta = \frac{3\pi}{2}$ radians

Arc length = $r\theta = \frac{39\pi}{2}$ cm

Perimeter = 2r + arc = $26 + \frac{39\pi}{2}$ cm

Answer: $26 + \frac{39\pi}{2}$ cm

4 Resultant force and acceleration (vectors) (AQA, Edexcel, OCR, MEI, Eduqas)

Two forces $(7\mathbf{i} + 7\mathbf{j})$ N and $(5\mathbf{i} - 2\mathbf{j})$ N act on a particle of mass 2 kg.

Find the magnitude of the acceleration of the particle.

Resultant = $(7\mathbf{i} + 7\mathbf{j}) + (5\mathbf{i} - 2\mathbf{j}) = 12\mathbf{i} + 5\mathbf{j}$

$|R| = \sqrt{(12^2 + 5^2)} = 13$ N

$a = F/m = 13 \div 2$

Answer: 6.5 m s^{-2}

5 Separable differential equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $\frac{dy}{dx} = \frac{6y}{x}$ for $x > 0$,

given that $y = 3$ when $x = 1$.

$\frac{1}{y}\,dy = \frac{6}{x}\,dx$, so $\ln y = 6 \ln x + c$

$x = 1$, $y = 3$: $c = \ln 3$

$y = 3x^6$

Answer: $y = 3x^6$

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6 Graph of the modulus of a quadratic (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the curve y = f(x), a quadratic.
(a) Find the coordinates of the local maximum point of y = |f(x)|.
(b) How many solutions has |f(x)| = 9?

(a) y = |f(x)| reflects the part of the curve below the x-axis in the x-axis:
(−3, −9) becomes (−3, 9)
(b) The maximum of the middle part is 9. Compare k = 9 with 9:
k = 9 gives 3 intersections with y = k.
Answer: (a) (−3, 9) (b) 3

7 Binomial critical region (All boards)

X ~ B(25, p). H_0: p = 0.2 is to be tested against H_1: p > 0.2 at the 5% significance level.
Find the critical region and the actual significance level.

Under H_0, X ~ B(25, 0.2)
P(X ≥ 9) = 0.0468 ≤ 0.05
P(X ≥ 8) = 0.1091 > 0.05
The critical region is X ≥ 9
Answer: X ≥ 9; actual significance level 4.68%

8 Projection from a height (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A ball is thrown vertically upwards at 4.9 m s^{-1} from a point 58.8 m above horizontal ground.
Take g = 9.8 m s^{-2}.
Find the time before it hits the ground.

Taking up as positive: $-58.8 = 4.9t - 4.9t^2$
$4.9t^2 - 4.9t - 58.8 = 0$
$t = (4.9 \pm \sqrt{1176.49}) \div 9.8 = (4.9 \pm 34.3) \div 9.8$
Positive root only.
Answer: 4 s

9 Venn diagram with independent events (All boards)

The Venn diagram shows probabilities for events A and B, which are independent.
Find the values of x and y.

P(A) = 0.15 + 0.1 = 0.25 and P(A ∩ B) = 0.1
Independent: P(A ∩ B) = P(A)P(B), so P(B) = 0.1 ÷ 0.25 = 0.4
x = P(B) − P(A ∩ B) = 0.4 − 0.1 = 0.3
y = 1 − (0.15 + 0.1 + 0.3) = 0.45
Answer: x = 0.3, y = 0.45

10 Chord of a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line y = 2x − 2 meets the circle $(x + 3)^2 + (y + 3)^2 = 25$ at the points A and B. Find the coordinates of A and B, and the exact length of the chord AB.

Substitute y = 2x − 2: $(x + 3)^2 + (2x + 1)^2 = 25$
$x^2 + 2x - 3 = 0$
(x − 1)(x + 3) = 0, so x = −3 or x = 1
y = 2x − 2: A = (−3, −8), B = (1, 0)
$AB^2 = 4^2 + 8^2 = 80$
Answer: A(−3, −8), B(1, 0), AB = $4\sqrt{5}$