Day 188 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Solving exponential equations using logarithms (All boards)

Solve $e^{5x} = 34$, giving your answer to 3 s.f.

Take natural logs: $5x = \ln 34$

$x = \frac{\ln 34}{5} = 0.7053$

Answer: $x = 0.705$

2 Geometric series and logarithms (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A geometric series has first term 8 and common ratio 2.
Find the smallest value of n for which the sum of the first n terms exceeds 10000.

$S_n = 8(2^n - 1) > 10000$

$2^n > 1251$

$n > \frac{\lg 1251}{\lg 2} = 10.289$

So the smallest integer n is 11

Answer: $n = 11$

3 Product rule (All boards)

Given that $y = (x + 2)e^{-2x}$,

find $\frac{dy}{dx}$, and its value when $x = 0$.

Product rule with $u = x + 2$, $v = e^{-2x}$

$u' = 1$, $v' = -2e^{-2x}$

$\frac{dy}{dx} = u'v + uv' = (-2x - 3)e^{-2x}$

When $x = 0$: $\frac{dy}{dx} = -3$

Answer: $\frac{dy}{dx} = (-2x - 3)e^{-2x}$; value -3

4 Unknown force from acceleration (vectors) (AQA, Edexcel, OCR, MEI, Eduqas)

A particle of mass 3 kg is acted on by forces $(-6\mathbf{i} - 3\mathbf{j})$ N and $\mathbf{P}$ N only. Its acceleration is $(3\mathbf{i} + \mathbf{j})$ m s^{-2}.
Find $\mathbf{P}$, and its magnitude to 3 significant figures.

F = ma: $(-6\mathbf{i} - 3\mathbf{j}) + \mathbf{P} = 3(3\mathbf{i} + \mathbf{j}) = (9\mathbf{i} + 3\mathbf{j})$

$\mathbf{P} = 15\mathbf{i} + 6\mathbf{j}$

$|\mathbf{P}| = \sqrt{(15^2 + 6^2)}$

Answer: $\mathbf{P} = (15\mathbf{i} + 6\mathbf{j})$ N; $|\mathbf{P}| = 16.2$ N

5 Proof using parity (AQA, Edexcel, OCR, MEI, Eduqas)

Prove that if n is odd then $n^2 + 5n + 6$ is even.

Let $n = 2k + 1$, where k is an integer.

$n^2 = 4k^2 + 4k + 1$

$5n = 10k + 5$

Total $= 4k^2 + 14k + 12$

$4k + 14$ is even and 12 is even, so the total is even.

Answer: the expression is even

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6 Horizontal projection from a height (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A stone is thrown horizontally at 13 m s^{-1} from the top of a cliff 44.1 m above level ground.
Take g = 9.8 m s^{-2}.
Find the time of flight and the horizontal distance from the base where it lands.

Vertical: $44.1 = \frac{1}{2} \times 9.8 \times t^2$, $t^2 = 9$
t = 3 s
Horizontal: x = 13 × 3
Answer: 3 s, 39 m

7 Tree diagram (All boards)

The tree diagram shows probabilities for event E, followed by event F.
Find the probability that exactly one of E and F occurs.

P(E and not F) = 0.2 × 0.4 = 0.08
P(not E and F) = 0.8 × 0.4 = 0.32
Exactly one: 0.08 + 0.32 = 0.4
Answer: 0.4

8 Equations with sec, cosec and cot (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $2\tan^2 x - 3\sec x - 3 = 0$ for $0 \leq x \leq 360°$,
giving answers to 1 d.p.

Use $\tan^2 x = \sec^2 x - 1$: $2\sec^2 x - 3\sec x - 5 = 0$
$(2\sec x - 5)(\sec x + 1) = 0$, so $\sec x = \frac{5}{2}$ or -1
$\sec x = \frac{5}{2}$, so $\cos x = \frac{2}{5}$: x = 66.4°, 293.6°
$\sec x = -1$, so $\cos x = -1$: x = 180.0°
Answer: x = 66.4°, 180.0°, 293.6°

9 Binomial critical region (All boards)

X ~ B(40, p). H_0: p = 0.25 is to be tested against H_1: p < 0.25 at the 1% significance level.
Find the critical region and the actual significance level.

Under H_0, X ~ B(40, 0.25)
$P(X \leq 3) = 0.0047 \leq 0.01$
$P(X \leq 4) = 0.0160 > 0.01$
The critical region is $X \leq 3$
Answer: $X \leq 3$; actual significance level 0.47%

10 Chord of a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line y = 3x + 7 meets the circle $(x + 3)^2 + (y - 3)^2 = 65$ at the points A and B. Find the coordinates of A and B, and the exact length of the chord AB.

Substitute y = 3x + 7: $(x + 3)^2 + (3x + 4)^2 = 65$
$x^2 + 3x - 4 = 0$
$(x - 1)(x + 4) = 0$, so $x = -4$ or x = 1
y = 3x + 7: A = (−4, −5), B = (1, 10)
$AB^2 = 5^2 + 15^2 = 250$
Answer: A(−4, −5), B(1, 10), AB = $5\sqrt{10}$