Day 187 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Free fall from rest (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A coin is dropped from rest from a window, 19.6 m above the ground.
Take g = 9.8 m s^{-2}.
Find (a) the time taken to reach the ground, (b) its speed on landing.

(a) $s = ut + \frac{1}{2}at^2$: $19.6 = \frac{1}{2} \times 9.8 \times t^2$
$t^2 = 19.6 \div 4.9 = 4$, so $t = 2$
(b) $v = u + at = 9.8 \times 2$
Answer: (a) 2 s (b) 19.6 m s^{-1}

2 Simplifying surds (AQA, Edexcel, OCR, MEI, Eduqas)

Expand and simplify $(1 + 2\sqrt{3})(4 - 3\sqrt{3})$, giving your answer in the form $a + b\sqrt{3}$.

$(1 + 2\sqrt{3})(4 - 3\sqrt{3}) = 4 - 3\sqrt{3} + 8\sqrt{3} - 6\sqrt{3}\times\sqrt{3}$
$\sqrt{3}\times\sqrt{3} = 3$, so the expression $= -14 + 5\sqrt{3}$
Answer: $-14 + 5\sqrt{3}$

3 Inverse normal (All boards)

A battery lasts X hours, normal with mean 50, s.d. 3.
Find k (3 s.f.): (a) P(X < k) = 0.25,
(b) P(X > k) = 0.1.

(a) $z = -0.6745$, so $k = 50 + (-0.6745) \times 3 = 48.0$
(b) P(X < k) = 0.9, $z = 1.282$, so $k = 50 + 1.282 \times 3 = 53.8$
Answer: (a) 48.0 (b) 53.8

4 Newton-Raphson method (AQA, Edexcel, OCR, MEI, Eduqas)

$f(x) = e^x - 4x$. Use the Newton-Raphson method with
$x_0 = 0.4$ to find x_1 and x_2, to 4 d.p.

$f'(x) = e^x - 4$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_1 = 0.4 - \frac{-0.1082}{-2.508} = 0.3569$$

$$x_2 = 0.3569 - \frac{0.001368}{-2.571} = 0.3574$$

Answer: $x_1 = 0.3569$, $x_2 = 0.3574$

5 Particle in equilibrium: string and horizontal force (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle of mass 5 kg hangs in equilibrium on a light string inclined at α to the vertical, held by a horizontal force P N, as shown. $\tan\alpha = \frac{12}{5}$.

Take g = 9.8 m s^{-2}.
Find the tension in the string and the value of P.

$\cos\alpha = 5/13$, $\sin\alpha = 12/13$
Vertical: $T\cos\alpha = mg = 49$, so $T = 49 \times 13/5$
Horizontal: $P = T\sin\alpha = 127.4 \times 12/13$
Answer: T = 127.4 N; P = 117.6 N

6 Equations with sec, cosec and cot (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $4\tan^2 x - 4\sec x - 11 = 0$ for $0 \leq x \leq 360°$,
giving answers to 1 d.p.

Use $\tan^2 x = \sec^2 x - 1$: $4\sec^2 x - 4\sec x - 15 = 0$

$(2\sec x - 5)(2\sec x + 3) = 0$, so $\sec x = \frac{5}{2}$ or $-\frac{3}{2}$

$\sec x = \frac{5}{2}$, so $\cos x = \frac{2}{5}$: $x = 66.4°, 293.6°$

$\sec x = -\frac{3}{2}$, so $\cos x = -\frac{2}{3}$: $x = 131.8°, 228.2°$

Answer: $x = 66.4°, 131.8°, 228.2°, 293.6°$

7 Coefficient in a product with a binomial (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the coefficient of x^3 in the expansion of
$(1 + 3x)(2 + 2x)^4$.

$(2 + 2x)^4$ has x^2 coefficient 96 and x^3 coefficient 64

x^3 term of the product $= 1 \times 64 + 3 \times 96$

$= 352$

Answer: 352

8 Hypothesis test for correlation (AQA, Edexcel, OCR, MEI, Eduqas)

15 pairs of hours of sleep and reaction time give $r = 0.547$.
Two-tailed critical value (5%): 0.5140.
Test for correlation.

$H_0: \rho = 0$, $H_1: \rho \neq 0$

$|r| = 0.547 > 0.5140$

r is in the critical region, so reject H_0

Conclusion: there is evidence of correlation between hours of sleep and reaction time

Answer: reject H_0: there is evidence of correlation between hours of sleep and reaction time

9 R cos(θ − α) form: express and maximum (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Express $5\cos\theta + 12\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < 90°$. Give α to 1 d.p.

$R\cos\alpha = 5$, $R\sin\alpha = 12$

$R = \sqrt{5^2 + 12^2} = 13$

$\tan\alpha = \frac{12}{5}$, so $\alpha = 67.4°$

Answer: $13\cos(\theta - 67.4°)$

10 Chord of a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line $y = -x - 2$ meets the circle $(x - 2)^2 + (y - 1)^2 = 25$ at the points A and B. Find the coordinates of A and B, and the exact length of the chord AB.

Substitute $y = -x - 2$: $(x - 2)^2 + (-x - 3)^2 = 25$

$x^2 + x - 6 = 0$

$(x - 2)(x + 3) = 0$, so $x = -3$ or $x = 2$

$y = -x - 2$: A = (−3, 1), B = (2, −4)

$AB^2 = 5^2 + (-5)^2 = 50$

Answer: A(−3, 1), B(2, −4), AB = $5\sqrt{2}$