Day 186 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Newton-Raphson method (AQA, Edexcel, OCR, MEI, Eduqas)

$f(x) = e^x - 3x$. Use the Newton-Raphson method with $x_0 = 0.6$ to find x_1 and x_2, to 4 d.p.

$f'(x) = e^x - 3$

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$x_1 = 0.6 - \frac{0.02212}{-1.178} = 0.6188$

$x_2 = 0.6188 - \frac{0.0003233}{-1.143} = 0.6191$

Answer: $x_1 = 0.6188$, $x_2 = 0.6191$

2 Centre and radius of a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A circle has equation $x^2 + y^2 - 8x - 10y + 37 = 0$.
Find the coordinates of its centre and its radius.

Complete the square in x and y:

$(x - 4)^2 - 16 + (y - 5)^2 - 25 + 37 = 0$

$(x - 4)^2 + (y - 5)^2 = 4$

Centre (4, 5), radius 2

Answer: centre (4, 5), radius 2

3 Mean and standard deviation from sums (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A summary of the ages, a years, of 9 members of a club gives $\Sigma a = 212$ and $\Sigma a^2 = 6670$.
Find the mean and the standard deviation (3 s.f.).

Mean $= 212 \div 9 = 23.6$

Variance $= \frac{6670}{9} - (23.556)^2 = 186.2$

Standard deviation $= \sqrt{186.2} = 13.6$

Answer: mean 23.6, standard deviation 13.6

4 Inequalities with exponents and logs (All boards)

Solve $7^x > 288$, giving your answer to 3 s.f.

Take logs: $x \ln 7 > \ln 288$

$x > \frac{\ln 288}{\ln 7} = 2.910$

Answer: $x > 2.91$

5 Equation of a circle from given information (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A circle has centre $(-4, 1)$ and passes through the point $(-4, 3)$.
Find an equation of the circle.

$r^2 = (-4 - (-4))^2 + (3 - 1)^2$

$= 0 + 4 = 4$

Answer: $(x + 4)^2 + (y - 1)^2 = 4$

6 Laws of logarithms: solving equations (All boards)

Solve $\log_2(x + 20) - \log_2(x - 1) = 2$.

$\log_2(\frac{x + 20}{x - 1}) = 2$, so $\frac{x + 20}{x - 1} = 2^2 = 4$

$x + 20 = 4(x - 1) = 4x - 4$

$3x = 24$, so $x = 8$

Answer: $x = 8$

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7 Velocity parallel to a direction (AQA, Edexcel, OCR, MEI, Eduqas)

A particle moves so that at time t s its velocity is $\mathbf{v} = (t^2 + 3)\mathbf{i} + (2t - 6)\mathbf{j}$ m s^{-1}.
Find the time when the particle is moving in the direction of $\mathbf{i}$, and its speed then.

Parallel to $\mathbf{i}$: $\mathbf{j}$ component = 0
$2t - 6 = 0$, so $t = 3$
$\mathbf{v} = (1 \times 3^2 + 3)\mathbf{i} = 12\mathbf{i}$
Answer: t = 3 s, speed 12 m s^{-1}

8 Distribution of the sample mean (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The time, X minutes, to complete a task is normally distributed with mean 150 and s.d. 5.
A random sample of 25 is taken. Find the probability that the mean time is more than 152 (3 s.f.).

$\overline{X} \sim N(150, \frac{5^2}{25})$, standard deviation 1.000
$z = \frac{152 - 150}{1.000} = 2.000$
P = 0.0228
Answer: 0.0228

9 Factor theorem with an unknown (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = 2x^3 + kx^2 + 4x - 16$.
Given that $(x - 1)$ is a factor of f(x),
(a) find the value of k,
(b) factorise f(x) completely.

f(1) = 0: 2 + k + 4 − 16 = 0
k = 10
$f(x) = (x - 1)(2x^2 + 12x + 16)$
$= 2(x - 1)(x + 2)(x + 4)$
Answer: (a) k = 10 (b) f(x) = 2(x − 1)(x + 2)(x + 4)

10 Rod suspended by two vertical strings (AQA, Edexcel, OCR, MEI, Eduqas)

A uniform rod AB of length 4 m and mass 5 kg hangs horizontally from two vertical strings attached 1 m from A and 1 m from B.
A particle of mass 5 kg is attached 1.5 m from A. Take g = 9.8 m s^{-2}. Find the tension in each string.

Moments about the string near A: $T_2(2) = 5\,g(1) + 5\,g(0.5)$
$T_2 = 36.75$
$T_1 + T_2 = 10\,g$ so $T_1 = 98 - 36.75$
Answer: Near A: 61.25 N; near B: 36.75 N