Day 185 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Equation of a line through two points (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The points A and B have coordinates (5, −4) and (1, −6).
(a) Find the midpoint of AB.
(b) Find the line AB as ax + by + c = 0, with integer a, b, c.

(a) Midpoint = $(\frac{5+1}{2}, \frac{-4+(-6)}{2}) = (3, -5)$

(b) Gradient = $\frac{-2}{-4} = \frac{1}{2}$

$y + 4 = \frac{1}{2}(x - 5)$

Multiply out and collect terms: $x - 2y - 13 = 0$

Answer: (a) (3, −5) (b) $x - 2y - 13 = 0$

2 Chain rule (All boards)

Given that $y = \sqrt{4x - 15}$,
find $\frac{dy}{dx}$, and the gradient of the curve when x = 4.

Chain rule: differentiate the outer function, then multiply by the derivative of the inner function

$\frac{dy}{dx} = \frac{2}{\sqrt{4x - 15}}$

When x = 4: gradient = 2

Answer: $\frac{dy}{dx} = \frac{2}{\sqrt{4x - 15}}$; gradient 2

3 Discrete random variable (AQA, OCR, MEI, Eduqas, CIE)

$P(X = x) = kx^2$ for x = 1, 2, ..., 3.
(a) Find k.
(b) Find E(X) and Var(X).

(a) $k(1 + 4 + 9) = 1$, so $14k = 1$ and $k = \frac{1}{14}$

(b) $E(X) = 1 \times \frac{1}{14} + 2 \times \frac{2}{7} + 3 \times \frac{9}{14} = \frac{18}{7}$

$E(X^2) = 7$, so $Var(X) = 7 - (\frac{18}{7})^2 = \frac{19}{49}$

Answer: (a) $k = \frac{1}{14}$ (b) $E(X) = \frac{18}{7}$, $Var(X) = \frac{19}{49}$

4 Describing transformations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Describe the single transformation that maps the graph of y = f(x) onto the graph of:
(a) y = f(−x)
(b) y = f(x − 5)
(c) $y = f(\frac{x}{4})$

(a) y = f(−x) is a reflection in the y-axis.

(b) y = f(x − 5) is a translation $\begin{pmatrix} 5 \\ 0 \end{pmatrix}$.

(c) $y = f(\frac{x}{4})$ is a stretch, scale factor 4, parallel to the x-axis.

Answer: (a) reflection in the y-axis (b) translation $\begin{pmatrix} 5 \\ 0 \end{pmatrix}$ (c) stretch, scale factor 4, parallel to the x-axis

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5 Independent and mutually exclusive events (All boards)

A and B are mutually exclusive events with P(A) = 0.2 and P(A ∪ B) = 0.4.
(a) Find P(B).
(b) Find P(A' ∩ B').

(a) Mutually exclusive: P(A ∪ B) = P(A) + P(B)
P(B) = 0.4 − 0.2 = 0.2
(b) P(A' ∩ B') = 1 − P(A ∪ B) = 0.6
Answer: (a) 0.2 (b) 0.6

6 Vertical projection upwards (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A stone is projected vertically upwards with speed 18 m s^{-1} from ground level.
Take g = 9.8 m s^{-2}. Find, to 3 significant figures,
(a) the greatest height reached, (b) the time of flight.

(a) $0 = 18^2 - 2 \times 9.8 \times H$, so $H = 324 \div 19.6$
(b) s = 0: $t = 2 \times 18 \div 9.8$
Answer: (a) 16.5 m (b) 3.67 s

7 Separable differential equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find y in terms of x, given that $\frac{dy}{dx} = 5y^2e^x$
and y = 0.25 when x = 0.

$\frac{1}{y^2}\,dy = 5e^x\,dx$, so $-\frac{1}{y} = 5e^x + c$

x = 0, $y = \frac{1}{4}$: $-4 = 5 + c$, so $c = -9$

$\frac{1}{y} = 9 - 5e^x$, so $y = \frac{1}{9 - 5e^x}$

Answer: $\mathbf{y = \frac{1}{(9 - 5e^x)}}$

8 Modulus inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve |3x + 5| < 4.

|3x + 5| < 4 means $-4 < 3x + 5 < 4$

Subtract 5 and divide by 3: $-3 < x < -\frac{1}{3}$

Answer: $\mathbf{-3 < x < -\frac{1}{3}}$

9 Intersection of two lines in 3D (CIE)

Line l_1: $\mathbf{r} = \begin{pmatrix} 6 \\ -3 \\ 7 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$.

Line l_2: $\mathbf{r} = \begin{pmatrix} 1 \\ -2 \\ -3 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$.

Show that l_1 and l_2 intersect and find the point.

i: $6 + 1\lambda = 1 + 1\mu$
j: $-3 + (-2)\lambda = -2 + 1\mu$
Solving i and j: $\lambda = -2$, $\mu = 3$
k check: $7 + 2(-2) = 3$ and $-3 + 2(3) = 3$, equal
Point: $\begin{pmatrix} 4 \\ 1 \\ 3 \end{pmatrix}$
Answer: $\mathbf{\begin{pmatrix} 4 \\ 1 \\ 3 \end{pmatrix}}$

10 Rod hinged at one end and held by a string (AQA, Edexcel, OCR, MEI, Eduqas)

A uniform rod AB of length 6 m and mass 24 kg is hinged at A and held horizontal by a light string attached at B. The string makes an angle θ with the rod, where $\sin\theta = \frac{3}{5}$. Take $g = 9.8$ m s^{-2}. Find the tension in the string.

Moments about A: $T \sin\theta \times 6 = mg \times 3$

$T \times \frac{3}{5} \times 6 = 235.2 \times 3$

$T = 705.6 \div 3.6$

Answer: 196 N